Leibniz's arithmetic machine

by Yves Serra, engineer

The history of calculating machines boasts two grand philosophers at its origin.

In 1642, Blaise Pascal presented an adding machine, later termed Pascaline;[1] from 1673 Leibniz took up the torch with the objective of manufacturing a multiplying machine which would free scientists from the duty of repetitive calculations.

> *thanks to my machine,* […] *the calculations could be successfully done by a little child.*

Leibniz (1646-1716) worked all his life on this project. He presented the first plans in 1673 in London, and subsequently in Paris. He designed various models, but it is the 1706 model, described in *Miscellanea Berolinensis*, 1710,[2] published by Leibniz in Berlin (he did similar in London and Paris), that we discuss here. A prototype of this machine is preserved in the State Library of Lower Saxony, Hanover, Germany.

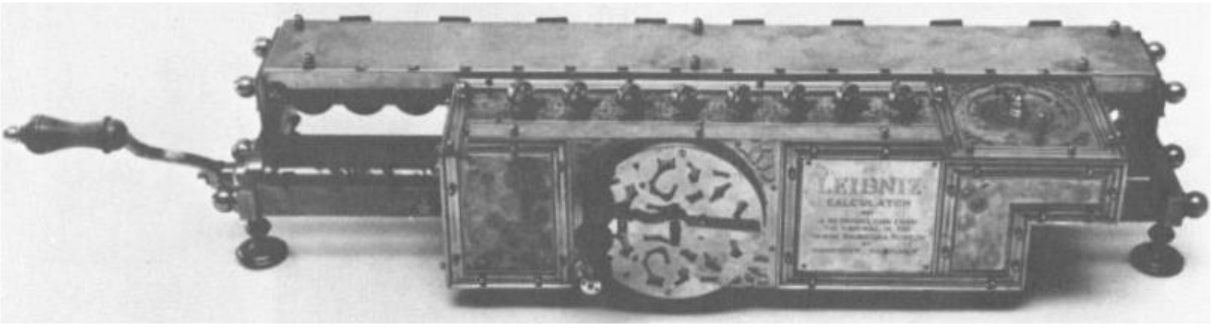


***Figure 1:* Photo of the original machine preserved in Hanover.**

Despite the significant effort, time and money that Leibniz invested in this project, the machine does not work correctly. The principles were sound and numerous innovations were implemented (we are going to review them by

1. See Pascal's text with commentary by D. Temam, BibNum, March 2009.
2. *Miscellanea Berolinensia ad Incrementum Scientiarum*: this magazine was created in 1710 and appeared until 1743; it succeeded *Histoire de l'Académie Royale des Sciences et des Belles-Lettres de Berlin.*

analysing the text), yet the smooth running of this machine required a level of mechanical engineering that was unattainable at the time. It was not until as late as the 19th century (around 1820) that Thomas de Colmar (1785-1870) constructed a reliable machine, drawing on Leibniz's ideas. Colmar's arithmometre was the first factory-made calculating machine. And it was only in the 20th century that researchers recreated a properly operating Leibniz machine:[3]

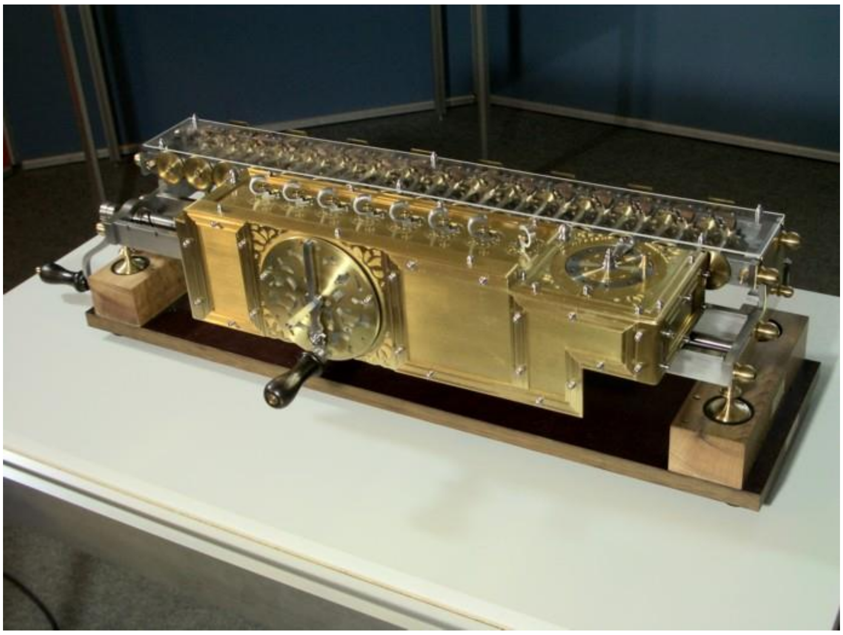


***Figure 2:* Replica constructed at Dresden University of Technology, 1998 - 2001.**

@@@@@@@

The text by Leibniz is primarily an instruction manual highlighting the objective of making multiplication a "children's game", to use his own words.

The text begins with conventional reference to the scholars whom Leibniz frequented and with whom he discussed his project.

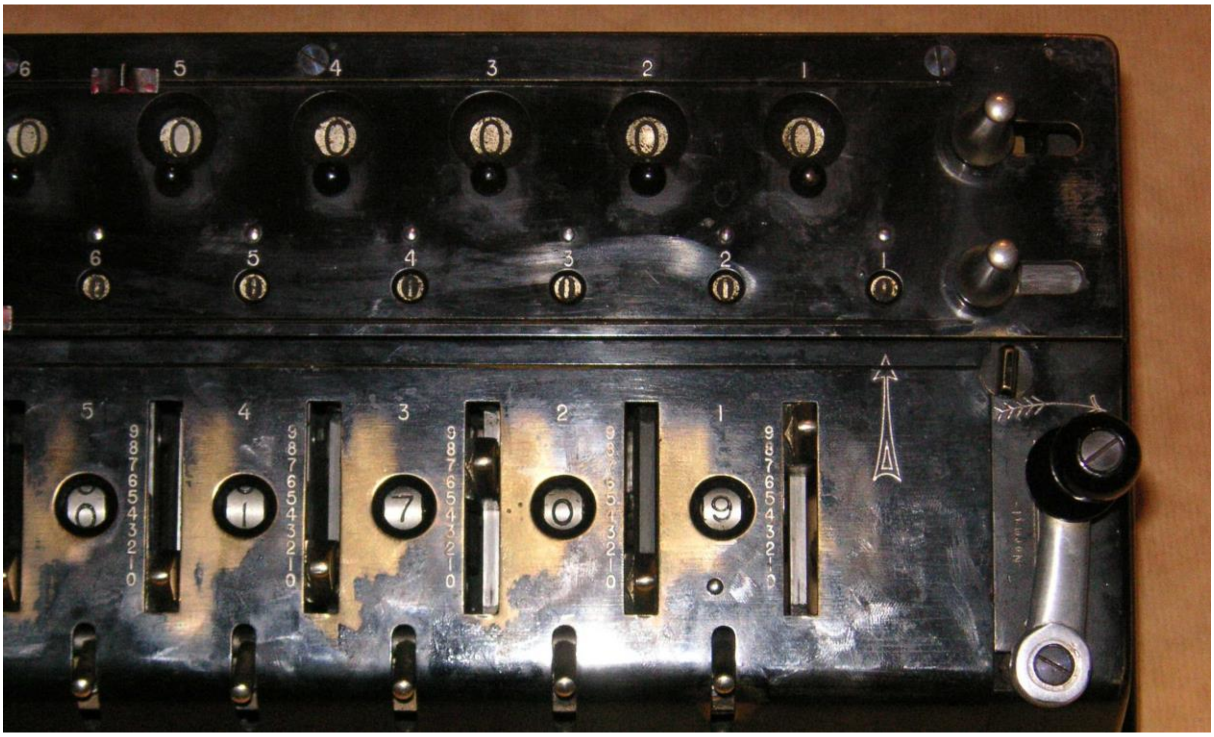
Among them we find names that are familiar to us. They include Antoine Arnaud, logician of Port Royal (1612-1694), a Jansenist theologian, nicknamed "The Great", and most notably the author of The General and Rational Grammar. There is also the physicist Christian Huygens (1629-1695) is referenced too, as is the mathematician and physicist Ehrenfried Walther von Tschirnhaus (1651-1708), who rediscovered in 1708 the way of making the china porcelain. There are others less familiar, such as Melchisédech Thévenot (1620-1692), traveller and guard of the king's library, whose fame does not seem to have reached us.

3. One may see the functioning of this replica in video recordings, at the University of Jena site.

Leibniz presents then the primary innovation of his machine at the beginning of the text. He says that it consists of one immobile part, and several that are mobile. The utility of this set-up is described in the text by the example of the multiplication of 1,709 (the year of writing) by 365 (the number of days in a year).

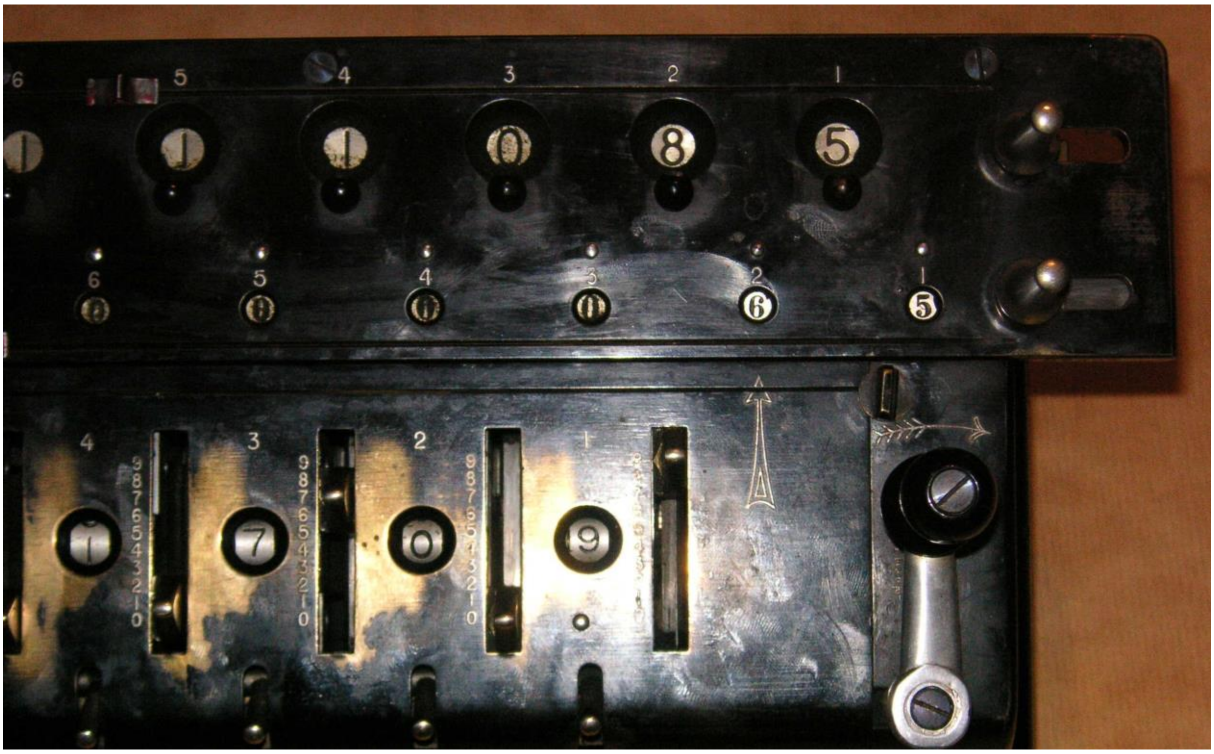
This combination has a double function. First, it allows the multiplicand (here 1,709) to be entered on the mobile part, independently of its transfer to the immobile part, which receives the intermediate results as a calculation is performed. This differs from a Pascaline, in which every number entered is transferred directly as a partial result.

Furthermore, this mechanical solution allows the mobile part to be shifted the left, to the power of 10 of the considered multiplier. To illustrate this device, we use a machine operating in the same way as that designed by Leibniz (the TIM machine of 1907, photos Yves Serra). To multiply 1,709 by 365, we first enter 1,709:

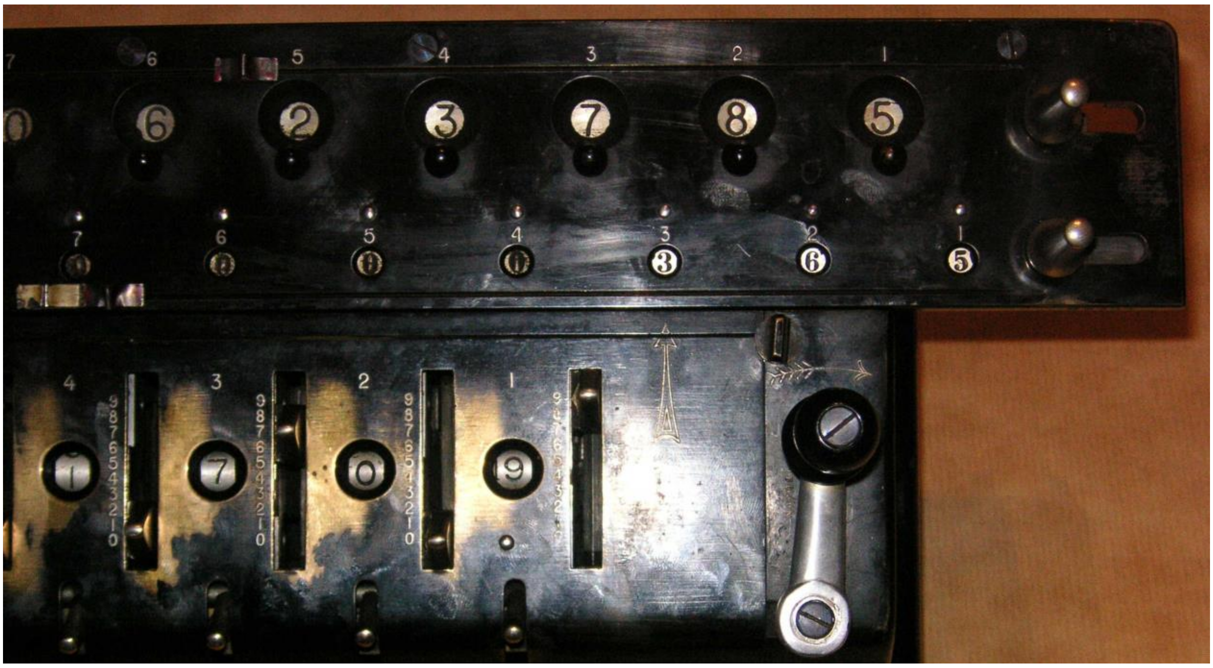


We multiply by five, by turning the crank five times. The digits of the multiplicand are in front of the digits of the results, the position indicated by the white arrow, the first partial result shows, 8,545 (namely five times 1,709):

Now we are going to shift the mobile part of the notch to the left (this can be seen on the right hand side of the photo below). The units of multiplicand are now in front of the tenths of the result, which we then multiply by six (by turning the crank six times), the new partial result, 111,085 (1,709 x 65) shows at the top:



Finally, we repeat the operation by shifting the whole mobile part of a notch to the left and multiplying by three, while the units of the multiplicand are in front of the hundredths of the final result, namely 623,785:



@@@@@@@

Leibniz's innovation, using mobile and immobile parts, will later be reapplied in calculating machines allowing multiplication, such as Colmar's arithmometre, which was the first factory-built and mass-produced machine of this type, and in its countless descendants, including the TIM machine in the photo above.

The second of Leibniz's innovations concerns the transfer of the multiplicand, here 1,709, from the mobile part to the immobile one, by rotating the crank. Hence it requires five rotations to multiply by five - the last digit of 365. The partial result that appears on the immobile part is 8,545.

Leibniz does not detail this second innovation in the text, which he intends to be instructions for use rather than a description of the machine's structure.

So let us present this innovation, with its famous "stepped drum of Leibniz with incremental teeth":

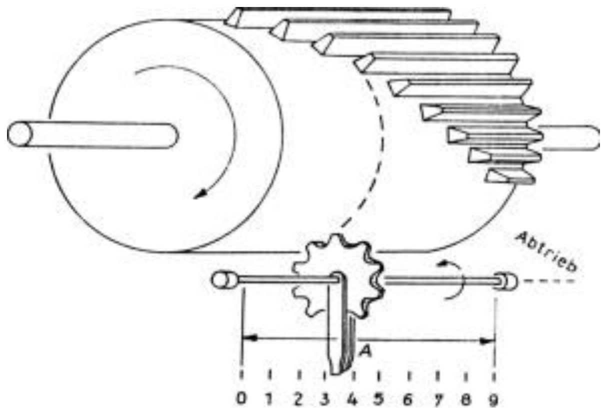


***Fig. 3:* The working of Leibniz's drum** *(University of Jena)*

Each figure of the multiplicand is registered by means of the pointer A, which, moving from left to right, will be in front of no teeth, then one tooth, then two teeth, up to nine. Such a cylinder is used for each of the powers of 10 of the multiplicand.

Entering 1,709 means making the lower thumb wheel of the first cylinder, that of the units, slide in front of nine teeth:

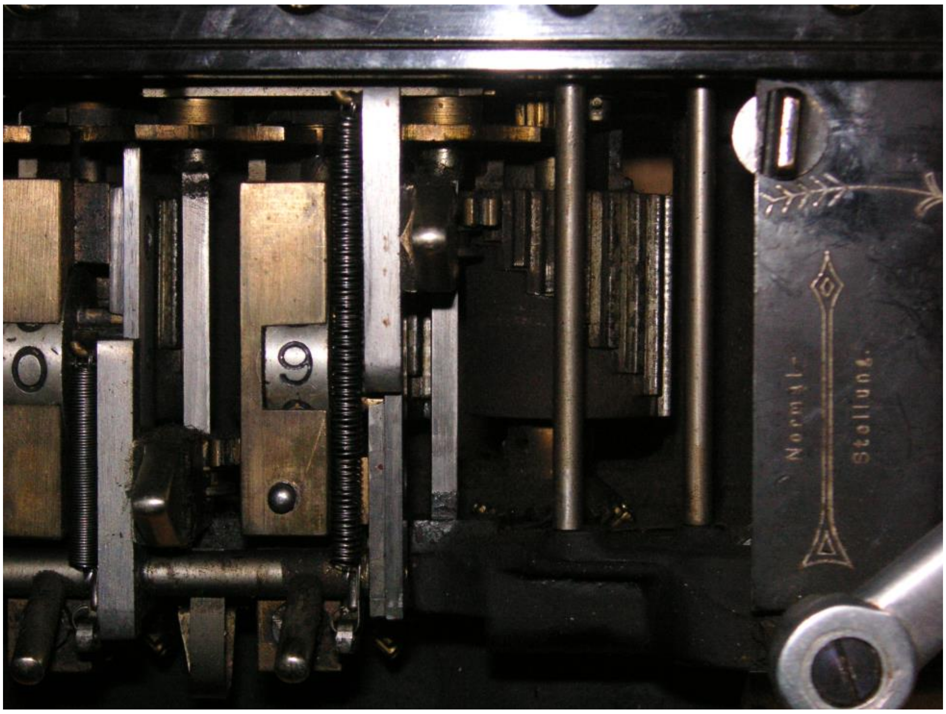


***Fig. 4:* Leibniz's drum in the TIM machine** *(photo Yves Serra)*

Then the second thumb wheel, that of the tens, moves to no teeth. The wheel for the hundreds rests in front of seven teeth, and that of the thousands in front of one tooth. Rotating the handle will cause four cylinders turn, and add to the partial result of the immobile part a value corresponding to each position. In this way, the multiplier is registered as the number of turns of the handle.

Adding or multiplying?

During addition, each operand is used only once. The simplest way to realise mechanical adding is when a movement that registers the operand adds the value to the result. This is how a Pascaline functions, where the gears feed the result as soon as the user inputs the figure to be added. It is also, of course, the way that an abacus functions, as well as many other more recent adding devices following Kummer’s adding device with stylus (1844).

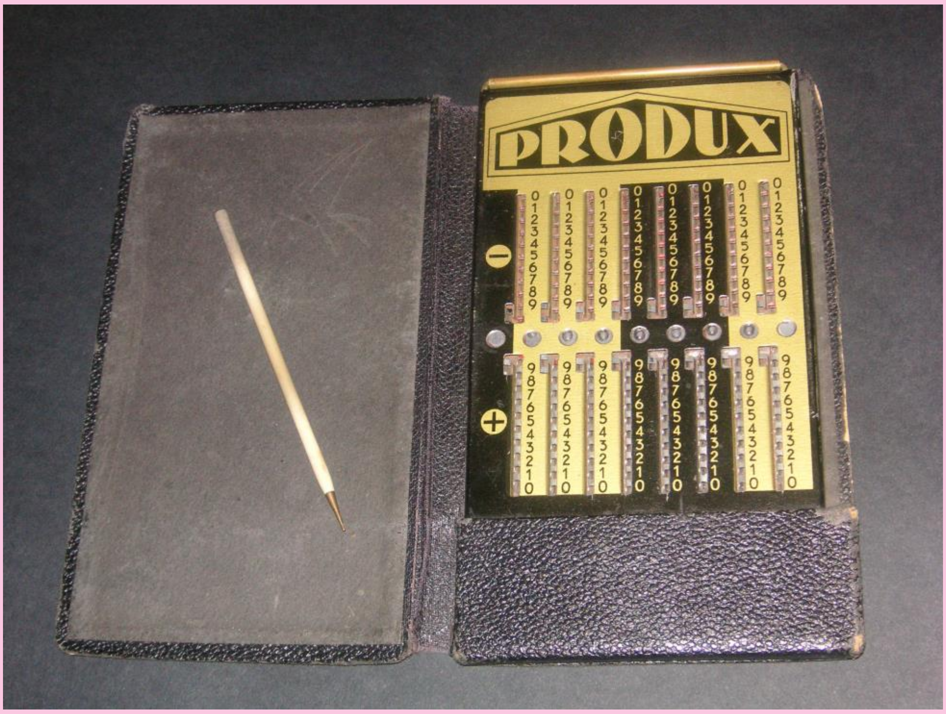


***Fig. 5:* Adding device of the Kummer type** *(photo Yves Serra)*

These machines are not adapted for multiplication. The only way to multiply, for example, 1,709 by five, would be to add 1,709 to 1,709, then adding 1,709 to the result three more times. Two solutions were designed to overcome this and create a multiplying machine deserving of the name. The first and more popular is the one proposed by Leibniz, which separates, on one hand, the fact of entering 1,709 (in today’s words, putting it in the memory), and on the other hand, using it five times in succession by simply rotating the handle. Thus the machine developed by Leibniz allowed multiplication by successive automatized adding. The second solution, little used, is direct multiplication based on referencing multiplication tables, of which an example is the renowned Millionaire calculator (patented in 1895).

Later, several devices came to compete with the Leibniz's cylinder, but this innovation was used until recently by mechanical calculating machines. Only in the middle of the 1970s were they replaced with electronic models.

La "Curta" - 1950 to 1970

Before the advent of the electronic calculator, machines based on the Leibniz type (with mobile and immobile parts) were used, especially in the domain of engineering sciences. The Curta (named after Curt Herzstark, who invented it during the Second World War) functioned thus, and its small size – no bigger than a peppermill - guaranteed its success. It was in production until the 1970s.

@@@@@@@

The text provides a true algorithm, describing every stage of the approach and showing the partial results that appear on the immobile part during calculation:

> […] *the result of 1,709 by five, namely 8,545, will appear on the extreme right of the immobile part in the openings. Given that the multiplier has many digits (three, six and five), and that the digit following five is six (…)*
>
> […] *not only will the multiplicand 1,709 be multiplied by six, but the result will also be added to the first result, and the number 111,085 will show in the openings located on the right of the immobile part.*

Leibniz insists on the great advantage of this method, that: the effort will be identical whatever the values to multiply may be, and the accuracy of the result is certain. A single limit is to be taken into account, and that is the maximal value of the display on the fixed part. We would refer today to this as the risk of memory capacity overflow.

Let us suppose the outcome of a multiplicand by a multiplier, and that the result does not exceed twelve figures

The presentation of division is only touched upon; it is made according to the same principle as multiplication, but by rotating the crank in the inverse direction, which leads to the value of the divisor being removed every time, the quotient being the number of turns of the crank and the remainder, temporary or final, appears as an intermediate result on the immobile part.

A simple turn of the crank in one direction (addition) or in the other (subtraction) allows what is recorded on the mobile part to be added to or subtracted from what appears on the immobile one. Thus, four operations are well within the reach of a child by the Leibniz machine.

@@@@@@@

To conclude, let us recall that Leibniz also presented a memoir on binary calculation to the Paris Academy of Science in 1703 (entitled 'Explanation of Binary Arithmetic, which uses only the Characters 0 and 1, with Some Remarks on Its Usefulness, and on the Light It Throws on the Ancient Chinese Figures of Fuxi'). Let us imagine how the history of calculation could have developed if he had combined these two reflections, in the beginning of the 18th century, and seen that binary calculation was the best candidate for the realisation of calculation, the development of which he had already so strongly contributed to with his machine.

■

(published August 2009)

(translated in English, published October 2015)

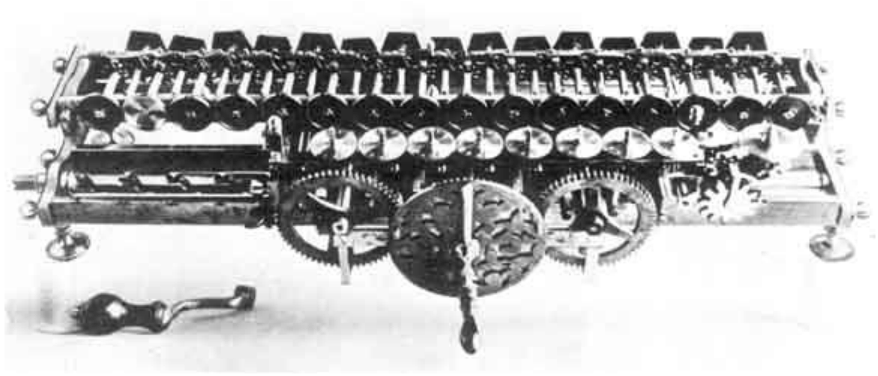


***Fig. 6:* Internal mechanism of the original Leibniz machine** *(The Hanover Library)*