# Coriolis: the Birth of a Force

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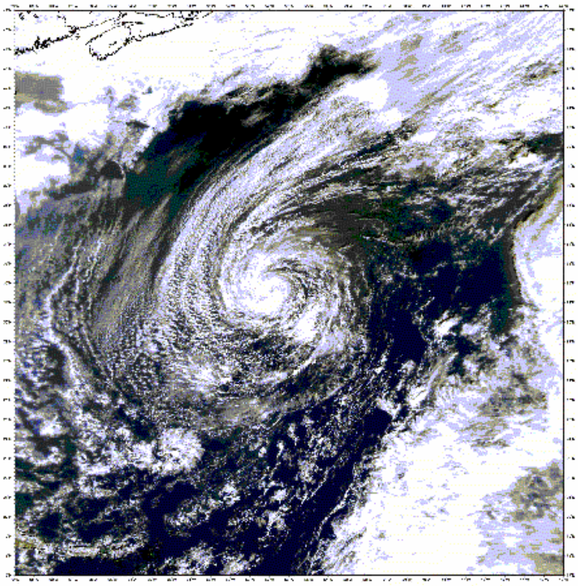
In one of his bestsellers, *Portrait du Gulf Stream* (Seuil 2005), novelist Erik Orsenna devotes a chapter to Gaspard-Gustave de Coriolis (1792-1843), concluding as follows: "There is no indication that our Gaspard Gustave ever set foot on a boat or was interested in the sea. Coriolis will always be the one who explained the influence of Earth's rotation on the route of winds and currents".



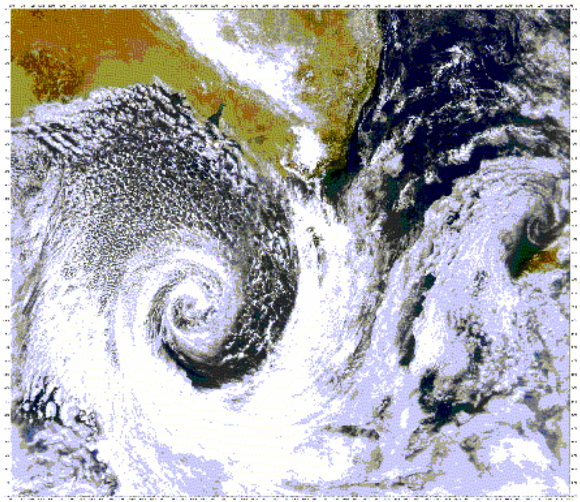


***Figure 1:* (on the left) A cyclone in the northern hemisphere** *(anticlockwise): Hurricane Olga, on 28th November 2001 in the Atlantic;* ***(on the right) A cyclone in the southern hemisphere*** *(clockwise): Australia, on 20th February 2002 - the south coast of Australia can be seen (images: NASA)*

This is how the name Coriolis became universally known; nevertheless, his career and other contributions are considerably less well-known, as well as how he came to 'compound centrifugal forces', to which his name would be given. This is what we will try to describe in this article, making sure to recall Coriolis's other contributions: he was the first person to give the physics definition of the term *work* in a paper presented in 1826 to the French 'Académie des Sciences' (although he was not yet a member). He was also the author of a real 'work theory' in his major and austere work, *Calcul de l'effet des machines* [*Calculation* *of the Effect of Machines*] in 1829 and he wrote a *Théorie mathématique des effets du jeu de billard* [*Mathematical Theory of the Game of Billiards*] in 1835, a rare example in the historiography of science in which a subject is totally handled,[1] leaving near enough no room for further possible contributions to be made on the matter.

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The birth of the 'Coriolis force' takes place in two other texts:

- "Mémoire sur le principe des forces vives dans les mouvements relatifs des machines" [On the Principle of Kinetic Energy in the Relative Movement of Machines], read on 6th June 1831 at Académie des sciences, published in *Journal de l'École polytechnique* in September 1832 (book 21, volume XIII) and in *Mémoires des savants étrangers*.

- "Mémoire sur les équations du mouvement relatif des systèmes de corps" [On the Equations of Relative Motion of a System of Bodies], published in *Journal de l'École polytechnique* in 1835 (book 24, volume XV).

The second paper mentions the 'compound centrifugal forces', which would later take the name 'Coriolis force' [or effect]. The first paper provides the calculation basis for the second result, also including the new notion at that time of 'inertial forces' (*forces d'entraînement*). This term remained even though Coriolis was not credited.

Another important difference between the two papers shows the evolution of Coriolis's work: the first, as we shall see, reveals a scalar identity focusing on the *vis viva* [or kinetic energy] in the relative motion (the *vis viva* proposed by Coriolis is $\frac{1}{2}mv^2$). The second reveals a more powerful and vector identity, focusing on the principle of dynamics in the relative motion. *A posteriori*, first paper's result becomes a simple case, particular to that of the second: by projecting the vector identity onto the curve of movement, the scalar identity of the first paper is obtained – since the compound centrifugal force 'does not work', its projection is zero in the direction of the movement.

1. This work followed player and former Napoleonic army officer Mingaud's 1820s invention of the 'queue à procédé', equipped with a hemispherical washer on the end allowing 'retro' effects; as the source of modern billiards games it profoundly changed the game – previously the cue had a square end which gave no effect.

### POISSON PRESENTS THE FIRST PAPER AT ACADÉMIE DES SCIENCES

A paper with an interesting approach to this "Mémoire sur le principe des forces vives dans les mouvements relatifs des machines" is Poisson's report from the meeting at the Académie des sciences, on 31st October 1831. As often happens, the rapporteur's text allows for better understanding of the text since it draws on that which seems essential and also puts the text into perspective.

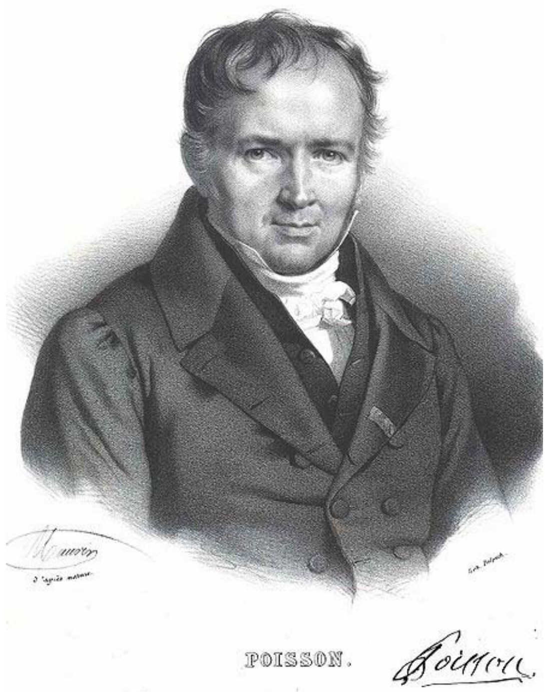


***Figure 2: Siméon-Denis Poisson (1781-1840).*** *École polytechnique student, author of numerous contributions in mathematics, rational mechanics and mathematical physics.*

Poisson begins by recalling the 'principle of live forces (*vis viva*)', as posed by d'Alembert:

$$mv_f^2 - mv_i^2 = 2\int_i^f \sum F dx \qquad (1)$$

The increase of 'live forces', between two successive positions of the system, is equal to twice the integral, taken between these limits, of the forces' sum with an effect on all these bodies multiplied each by the element of its direction; due to these forces the integral was at the time termed *amount of action*.

Poisson continues by indicating that Coriolis established the principle of live forces, not in the case of absolute but relative motion, for example inside a machine: on the left (*live forces* mv²) are the relative speeds involved in this frame; on the right (*amount of action*), the *inertial forces* must be subtracted.

This leads, in Coriolis's text, after four pages of precise calculations, to the following principle that he had written before formulating:

$$\frac{mV_r^2}{2} - \frac{mv_r^2}{2} = \int P\cos(\widehat{Pds_r})ds_r + \int P_e\cos(\widehat{P_e ds_r})ds_r \qquad (2)$$

*The equation contains this theorem, that the principle of live forces still takes place in the relative motion at moving axes, so long as at amounts of action* $\int P\cos(\widehat{Pds_r})ds_r$ *, calculated with the given forces P and the arcs dsr described in this relative motion, other amounts of action are added which result in forces Pe, are equal and opposite to those that ought to be applied to each moving point in order to make it take the motion that it would have if it were invariably linked to the moving axes.*

Let's compare the two equations *(1)* and *(2)*:

- Coriolis, as always, returns the factor 2 in the denominator of the left-hand side of the equation (while in the case of Poisson, in *(1)*, the multiplier is on the right side). Poisson, geometric follower of 'rational mechanics', only considered the 'live forces' defined by $mv^2$. As a teacher, Coriolis proposed to call *live forces* the quantities ½ $mv^2$: this is the meaning that will be the foundation of the later notion of *kinetic energy*.
- In equation *(2)*, the first term of the right-hand side corresponds to the right-hand side of equation *(1)*. But another term appears in equation *(2)* – which is that of the inertial forces. Considering the similarity of form of these two equations, it can be considered that Coriolis's equation *(2)* is a generalisation (as part of a relative movement) of the principle of Lagrange's live forces.

## ANALYSIS OF THE 1831 PAPER

This relatively complex paper has 20 pages and a six-paged appendix: *a posteriori* it appears only as a particular case of the second paper's result but it is larger and more calculative than the latter, making up 13 pages – actually, the second paper's results are latent in the first: in order to define the 'compound centrifugal force' Coriolis only has to resume half of his first paper's calculations.

From the introduction (part A), Coriolis begins by laying the groundwork for his result:

*...the equation of the live forces can be applied by entering the relative speeds, and the amounts of action or of work that also relate to relative motions. But in these amounts of action, also, forces that are immediately*

*given and that contribute at the absolute moment, others must be considered whose nature is easy to indicate: they are opposite the forces that ought to be applied to the material points of the system if they were free, to force them to conserve compared to the moving planes the relative positions that they have at some given point* [...]

But, from the outset, Coriolis warns against the statement's false evidence:

[...] *it would be a mistake if the proposal was regarded as obvious, even in this fairly simple example. It is so unobvious that these forces must be introduced that we would arrive at false results if we were to proceed with issues other than that of the live forces.*

Coriolis thus summarises the range of the paper, while anticipating, in a still unrecognised manner, the results of the second paper. The introduction of the 'inertial forces' - and of these only - is only valid for the principle of the live forces, and leads to the principle of the live forces in the relative motion, stated in *(2)* above (what René Dugas[2] would call 'Coriolis's first theorem').

Because any other equation of the relative motion requires, in addition to the inertial force, the introduction of the 'compound centrifugal force' (Coriolis's second theorem, 1935): as this 'does not work', the scalar projection in the direction of the movement makes it disappear, thereby leading to *(2).*

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Part (B) of the paper (p.272-277), after the introduction, is the broadest and most theoretical. Coriolis is placed in the case of two standardised yet non-orthogonal frames of reference, one for the fixed axes, another for the moving axes, and gives the relationships existing between the Cartesian coordinates of the same point in the two frames of reference in the most general way. Similarly, he expresses from a general outlook the links that exist during the movement $L = 0$, $M = 0$, etc., and uses Lagrange multipliers to project the expression of these liaisons on the moving axes. This generality is important for Coriolis, who insists (p.279): 'without specifying anything on these movements'.

This part leads to the main result of the text, 'Coriolis's first theorem', recalled in *(2)* above.

2. *Histoire de la mécanique*, Dunod, 1950 (reprint Jacques Gabay 1996) [see chap. IV, Mouvement relatif p. 354-367]

**Important Misprints in the *Journal de l'École Polytechnique***

Reading Coriolis's papers is quite difficult from a mathematical point of view, but also some misprints at crucial locations force the reader (either the current reader or of the time) to make rectifications to follow his reasoning:

- p. 273, bottom, last system of equations: replace x', y', z' with $x_1$, $y_1$, $z_1$.
- p. 274, below, penultimate system of equations: replace on the left side dx with $dx_1$; remove coefficients *m* in the three equations; in the second equation, right side, replace the second dy with dη.
- p. 274, below, last system of equations: replace on the left side $d_e^2x$ with $d_e^2x_1$ (idem for y, z).
- p. 279, equation (C): replace v with $v_r$.
- p. 279, last equation, remove the first *m*.
- p. 280, equation (D), *cos* is missing after $v_rv_e$.
- p. 281, concluding part of the equation, factor 2 is too much.
- p. 283, last equation, d²a instead of da.
- p. 284, last system of equations, many inconsistencies: replace (xdy − zdz) with (xdy − ydx), (ydz − xdy) with (ydz − zdy); only the second (zdx − xdz) is correct.
- p. 291, first equation, the clues *e* (for inertial force) are applied to the speed and not mass; *m* is missing in the second term.
- p. 291, second equation, mistake in the second term.

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Part (E) is the longest and the most difficult to access from the paper, because it does not seem to give any particular result compared to *(2)*. Coriolis details the last term of formula *(2)*, something new in the equation of the live forces, a term which is at the basis of Coriolis's first theorem: $\int P_e \cos(\widehat{P_e ds_r}) ds_r$ .

From the wording of his theorem he made sure to clarify:

> *We will indicate later how to represent these forces and simplify the calculations of quantities of action owed to them* [p. 277, just after the first theorem that introduces the inertial forces $P_e$].

> *Let's return to the calculation* $\int P_e\cos(P_e ds_r)ds_r$*, by no longer dealing withthe forces owed to the origin's movement of the moving axes* [p. 282]

Coriolis only mentioned eight pages later (p. 290) the possible movement of the origin, to immediately cancel his effect in practice (considering an unchanging rectilinear motion from the origin of mobile planes. This part (E) leads to the most general formula of the expression of the new term of the equation of the live forces, detailing this term that results from 'inertial forces':

$$\sum\int P_e \cos(\widehat{P_e ds_r}) ds_r = \sum \frac{mV_e^2}{2} - \sum \frac{mv_e^2}{2} - 2\sum m \int \frac{dN}{dt} d\sigma \qquad (3)$$

The term $\sum\int P_e \cos(\widehat{P_e ds_r}) ds_r$ making its appearance in the equation of the live forces of the relative movement in *(2)* is linked to two factors seen in *(3)*:

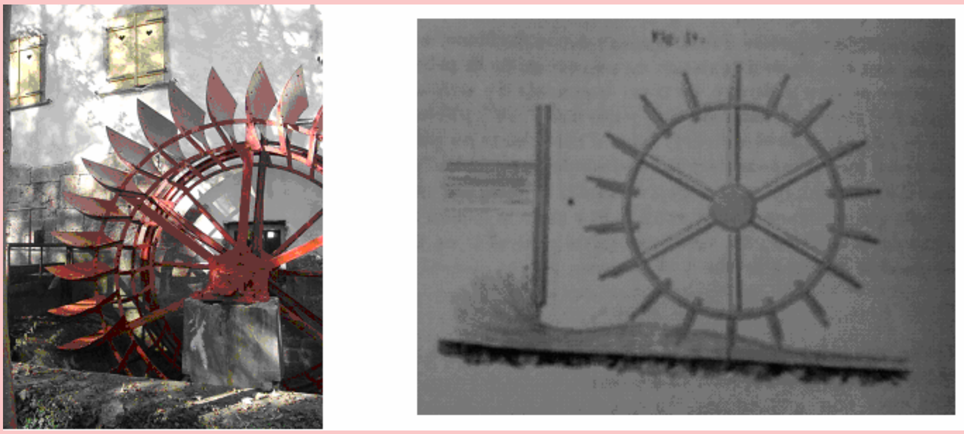
- The last term on the right represents the variation of the speed of rotation of the moving axes: when this is unchanging, this term is void. But Coriolis was keen to give the result in its greatest generality, indicated by the title of part (E) ("Expression générale de la quantité d'action qu'il faut introduire dans l'équation des forces vives, en raison du mouvement des axes mobiles" [General Expression of the Amount of Action that Must Be Introduced in the Equation of the Live Forces, Because of the Movement of the Moving Axes]).
- The first term (always on the right-hand side of the equation) is the difference of the live forces due to the difference of the inertial speeds over time. This part of the equation is indeed closer to the traditional principle of live forces *(1)*: the variation of an amount of action corresponds to the variation of the live forces linked to velocity, using this time the inertial velocity. Nevertheless, we cannot settle for this analogy with the traditional principle of the live forces, and a second factor must be added, which is the variation of the speed of the moving axes over time, represented by the second term.

Part (E) complements the first theorem: Coriolis aims to return to a known expression (the first part of *(3)*), and see the difference caused by his theorem, leading him to the equation's rightmost part *(3)*. Coriolis's concern is noticeable to get as close as possible to the traditional principle of live forces, and place himself, in form and substance, in the Lagrangian tradition of rational mechanics.

**Coriolis and the Poncelet Water Wheel**

Jean-Victor Poncelet (1788-1867), another Ecole polytechnique engineer-scholar, had received in 1825 the Mechanics prize from Académie des sciences for the invention of his water wheel (replacing the traditional type of wheel).

The major advantage of this invention, immediately adopted by manufacturers of wheel mills, was that water from the river or canal would enter tangentially at the blade (thanks to its curved shape): thus all water energy was of use in the blade, whereas in the flat-bladed wheel (to the right, below), water arrived with an angle of attack; there was shock against the blade and loss of energy – productivity was not optimal.



***<u>Figure 3:</u> The paddelwheel (Poncelet style) of the Lütterbach mill (Alsace), on the left; a 'bladed' wheel, extract from Poncelet's*** **l'Introduction à la mécanique industrielle,** ***on the right.***

Coriolis clearly does not dispute the usefulness of the curved blade compared to the flat blade. However, in his *Calcul de l'effet des machines* (1829), he had already found an initial mistake in the theory that Poncelet had made of his wheel[3]. In his 1831 article analysed here, Coriolis makes (p. 294) a second important objection, showing that similar theoretical productivity cannot be equal to 1, contrary to Poncelet's claim:

*The general formulas can be applied from the preceding articles to the issue of the movement of water in the curved blades of M. Poncelet's wheels. If we consider the blade like a canal where the water moves as it is lead in an unchanging rotational motion, the value of* $V_r$ *will be given in this case by* [...]

$$\Sigma \frac{mV_r^2}{2} = \Sigma \frac{mv_r^2}{2} + \Sigma \int P \cos(\widehat{P ds_r}) ds_r.$$

[...]*Following the usual form of curved blades, gravity will get closer to the direction of* $ds_r$*, that is to say, the tangent to the canal that forms the blade, more during water's descent than its ascent; it follows that* $\Sigma\, mV_r^2/2$ *is greater than* $\Sigma mv_r^2/2$*, and thus that the water leaves the wheel with a greater relative live force than what it had when entering.*

Put simply, the water encounters a weaker slope of the blade when it goes up than when it goes down, because in the meantime the wheel has turned: during the downward movement, the inclination of the blade is already closer to the vertical since it will soon release the water

3. This object was based on the fact that the model would not be known to be that of a single particle of water going up and down on the blade, but of a steady stream: thus 'the particles already risen, whose speed is less, inhibit the movement of those that are below and have more speed' (Coriolis, *Calcul de l'effet des machines*, 1829, § 105; for discussion of this point, *cf.* A.Moatti's thesis, 2011, *ibid.*, p.120)

(for a given particle of water, the blade's inclination on gathering the water [‘the water inlet’] is not the same as when releasing the water).
In Poncelet's theoretical calculation, the water enters with all its speed on the blade (thanks to the curve) and leaves with absolutely no speed. Coriolis corrects this last point indicating that in accordance with the work on gravity (the wheel having rotated), the absolute speed on leaving the blade is not zero – which was inevitable to the productivity of 1: if water keeps an absolute speed on releasing, this means that this speed (this ‘live force’) has not been used in the wheel. Carnot's principle tells us ‘that there is no such thing as a free lunch’ - no creation of live force: if water has an absolute speed that is not zero on leaving the blade, it carries a part of the live forces of the entire cycle…

## Analysis of the 1835 Paper

Paradoxically, the 1835 article “Mémoire sur les équations du mouvement relatif des systèmes de corps”, with the ‘Coriolis force’ result (‘Coriolis's second theorem’ to recover R. Dugas's expression), is smaller and less complex than the 1831 article. The second article's results are largely latent in the first article [4].

### Comparison with the First Paper

From the introduction of his second paper, Coriolis recalls the limits of his 1831 article, namely that it only applied to the principle of inertial forces – i.e., the <u>***scalar***</u> identity representing the motion's energy result - and not other equations of relative motion, like the fundamental principle of the dynamic $\boldsymbol{f} = m\gamma$, <u>***vector***</u> identity. Coriolis goes on to generalise the approach in his second paper, by asking the following questions: can we use the correction terms linked to inertial velocities in equations of movement other than the principle of live forces? If this is not the case, then can we ‘give a simple expression of the new terms of correction’? Coriolis answers these questions in his introduction:

> *To establish any equation of relative motion of a system of bodies or any machine, it is enough to add to existing strengths two species of additional forces; the former are always those that must be taken into account for the equation of live forces, that is to say that they are opposite forces to those that* […]*; the latter are managed perpendicularly at relative speeds and at the axis of rotation of the moving planes; they are equal to twice the product of the angular velocity of the moving planes multiplied by the amount of relative motion projected on a plane perpendicular to this axis.*

4. One can speculate that the second article, published in 1835 (or four years after the first, containing the seeds of later results), was written before 1835: the *Journal de l'Ecole polytechnique* allowed time to publish the articles that it received.

The paper from 1831 gave a scalar identity, which is that of the live forces. The 1835 paper gives a vector identity (which is a lot more powerful), applying to the vector laws of movement itself - like Newton's principle of dynamics. Coriolis shows immediately, in his introduction, that the 1831 result is a particular case of the second result, in other words it verifies the 1835 result in the particular case of the principle of live forces: indeed, the second correction terms (the 'Coriolis force $F_c$') being perpendicular to relative velocity, cosine $F_c$ *dsr* is zero; only remaining, in the rightmost side of the principle of the live forces, are the first terms of correction, which are those found in the first paper.

> *It is in this disappearance of these compound centrifugal forces that makes up the theorem that I presented to Académie des Sciences, in 1831. It now becomes a particular case of the more general statement about the introduction of these composed centrifugal forces.*

## Analysis of the Result and Comparison with the Modern Notations

Coriolis's demonstration is largely engaged in the 1831 paper. He uses an intermediate result of the first paper (p. 275) to lead very quickly to the second paper's result (p.146), while giving the equation of motion as follows:

$$m\frac{d^2x}{dt^2} = 2\left(rm\frac{dy}{dt} - qm\frac{dz}{dt}\right) + X - X_e + \lambda\frac{dL}{dx} + \mu\frac{dM}{dx} + \text{etc.} \qquad (5)$$

Let's analyse the different terms of this equation of motion, which is a vector equation in *x*, *y*, *z* (we only wrote the equation in *x*):

- The identity's leftmost term represents $m\gamma$, $\gamma$ being the acceleration in the moving frame of reference.
- The second rightmost term (X) represents the applied forces, measured in the moving frame of reference.
- The third rightmost term ($X_e$) represents the corrective term of the first paper, as defined by Coriolis (in fact the opposite of the 'inertial forces').
- Finally, the first rightmost term represents the opposite of the 'compound centrifugal force', where Coriolis gives three components on the moving axes:

$$2\left(rm\frac{dy}{dt}-qm\frac{dz}{dt}\right)$$
$$2\left(pm\frac{dz}{dt}-rm\frac{dx}{dt}\right)$$
$$2\left(qm\frac{dx}{dt}-pm\frac{dy}{dt}\right)$$

Here the precise expression of the coordinates of the Coriolis force is recognised, a force written in modern notations: $2m\vec{V}\wedge\vec{\Omega}$ (vector product), the vector **V** being the velocity vector in the moving frame of reference, of coordinates (dx/dt, dy/dt, dz/dt), the vector **Ω** being the angular velocity of rotation of the 'moving planes', of the coordinates (p, q, r).

Let's also note that the equation of motion [*(5)* above for the coordinate *x*, and the other two equations not shown for the coordinates *y* et *z*] constitutes a vector equation similar to the Newtonian equation of dynamics $\vec{F}=m\vec{\gamma}$. Coriolis rewrote a vector equation of relative motion, which is no longer $\vec{F}=m\vec{\gamma}$, but two corrected terms that are added, $\mathbf{F_e}$ the inertial force, and $\mathbf{F_c}$ the Coriolis force:

$$m\vec{\gamma}=\vec{F}+\vec{F_e}+\vec{F_c}$$

### The Designation of 'Compound Centrifugal Forces'

Coriolis uses the term 'compound centrifugal forces '(plural) to indicate the corrective term $\mathbf{F_c}$. It is interesting to see how and why Coriolis introduced this term, particularly because the notion of centrifugal forces is not used in the first paper. Coriolis's analogy is made from the opening of the second paper.

> *The latter have the greatest analogy with the ordinary centrifugal forces. To highlight this analogy, it is enough to note that the centrifugal force is equal to the amount of motion multiplied by the angular velocity of the tangent to the curve described, and that it is directed perpendicularly to speed and in the osculatory plane, i.e., also perpendicularly to the tangent's axis of rotation. So, to move from these ordinary centrifugal forces to the second forces whose doubles enter in the preceding statement, only the angular velocity of the tangent is replaced with that of the moving planes, as is the axis of rotation's direction of the tangent, with the direction of the axis of rotation of these same moving planes.*

If we try to translate this analogy into modern terms (and even though this analogy between centrifugal force and composed centrifugal force has now been completely abandoned in the presentation and teaching of the Coriolis force), the following comparison can be made:

- **Simple centrifugal force**: it has a scalar value **m**$\mathbf{V}\omega$ (amount of movement *mV* multiplied by angular velocity of the tangent $\omega$); it has a direction perpendicular to the velocity and 'to the axis of rotation of the tangent[5]'.
- **Compound centrifugal force** ($m\vec{V}\wedge\vec{\Omega}$): it has a scalar value **m**$\mathbf{V}\Omega$ (amount of movement mV multiplied by angular velocity of the moving planes $\Omega$[6]); it has a direction perpendicular to the velocity and 'to the axis of rotation of the moving planes'.

This is what lets Coriolis write, about the 'compound centrifugal forces':

> *These latter forces have the greatest analogy with the ordinary centrifugal forces.*

If we can explain the analogy with the centrifugal force, at any moment Coriolis explains the use of the term 'compound': one might think that the speed of rotation $\Omega$ is *compound* – via the vector product – with the moving **V**'s speed.

We will note in passing that Coriolis defines compound centrifugal force notion in which the double intervenes in his result, forcing him to speak each time of the 'double compound centrifugal forces'. One cannot miss that he had previously gone against the use of the term 'live forces' for $mv^2$, always obliging to speak of the only interesting physics quality, $\frac{1}{2}mv^2$, as 'half the live forces'.

> *We see that to move from ordinary centrifugal forces to second forces in which doubles enter in the equations of the relative motion, it is enough to replace at the same time, at the tangent's axis of rotation, the angular velocity, and the amount of motion of the moving point; the moving planes' axis of rotation, the angular speed of these planes, and the amount of motion projected onto a plane perpendicular to this axis. These second centrifugal forces* […] *can be called compound centrifugal forces.*

The modern meaning of the Coriolis force is 2m$\mathbf{V}\wedge\Omega$, with the coefficient 2; Coriolis himself did not take steps to include the coefficient 2 in what he refers to as 'compound centrifugal forces'.

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We will also note the difference between the titles of the two papers: the first covers only "le principe des forces vives dans les mouvements relatifs des

5. Let's take a material point in unchanging circular rotation around an axis perpendicular to the plane: the centrifugal force is orthogonal to the velocity and to the axis of rotation.
6. It is assumed here V and $\Omega$ are orthogonal, which does not involve the cosine of their angle.

machines”; the second on “les équations du mouvement relatif des systèmes de corps”. The term ‘equations of movement’ takes precedence over the term ‘principle of the live forces’, and generalises it; the term ‘relative movement of the system of bodies’ takes precedence over the term ‘relative movement of machines’, and generalises it. The impact of the second paper is a lot more general; as the plural disappeared in favour of the singular, the term ‘machines’ disappeared; and, in fact, the applications of the second paper will largely exceed the framework of the study of machines: the Foucault pendulum, meteorology, land geomagnetism involves the motion’s vector equations contained in the second paper – this while these applications involve neither the principle of the live forces nor any conservation of energy.

**Ampère (1830), Inertial Force and the Coriolis Force**

In an article[7] preceding those of Coriolis, and those which this one cites, Ampère highlights these two forces, but in a particular case (figure below) and without providing any interpretation to these forces.

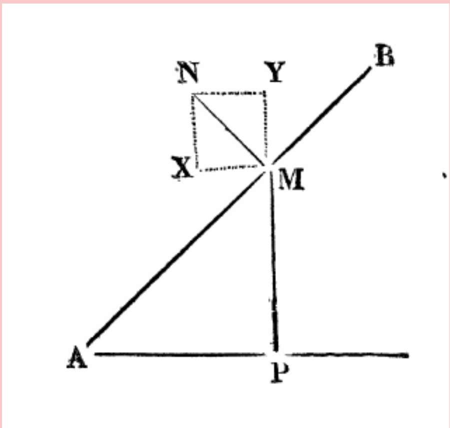



***Figure 4:*** *AB tube turns in a vertical plane around point A. Sphere M is moving in the tube (drawing from Ampère's 1830 article)*

We have $\overrightarrow{OM} = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j} = r\vec{u}$, $\dfrac{d\overrightarrow{OM}}{dt} = \dot{r}\vec{u} + r\dot{\theta}\vec{v}$ (v being the normal vector - sinθ i + cosθ j), deriving again (knowing that du/dt = θ’v, et dv/dt = - θ’u):

$$\vec{\gamma} = \frac{d^2\overrightarrow{OM}}{dt^2} = (\ddot{r}\vec{u} + \dot{r}\dot{\theta}\vec{v}) + (\dot{r}\ \dot{\theta}\vec{v} + r\ddot{\theta}\vec{v} - r\dot{\theta}^2\vec{u})$$

$$\vec{\gamma} = \ddot{r}\vec{u} + (r\ddot{\theta}\vec{v} - r\dot{\theta}^2\vec{u}) + 2\dot{r}\dot{\theta}\vec{v} \qquad (1)$$

The first term in r"u is the acceleration of the moving object in the true frame of reference in the tube.

7. ‘Dynamique. Solution d’un problème de dynamique, suivie de considérations générales sur le problème des forces centrales’, *Annales de mathématiques pures et appliquées*, 20 (1829-1830), p. 37-58.

The second term refers to the 'Coriolis's first theorem' (1831) - it is the inertial acceleration. We take point N located in M although indissoubly linked to the tube (r'=0). We have

$$\overrightarrow{ON} = r\vec{u}\text{, then } \overrightarrow{v_N} = r\dot{\theta}\vec{v}\text{, and } \overrightarrow{\gamma_N} = r\ddot{\theta}\vec{v} - r\dot{\theta}^2\vec{u}$$

The third term in *(1)* corresponds to the acceleration of Coriolis[8] applied to the moving object M in its own frame of reference:

$$\overrightarrow{\gamma_c} = 2\vec{\Omega} \wedge \vec{v} = 2\begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix} \wedge \begin{pmatrix} \dot{r} \\ 0 \\ 0 \end{pmatrix} = 2\dot{r}\dot{\theta}\vec{v}$$

Formula *(1)* therefore brings us to the principle of the live forces with respect to relative motion (identical to Coriolis 1835):

$$\overrightarrow{\gamma_{R'}} = \overrightarrow{\gamma_R} + \overrightarrow{\gamma_e} + \overrightarrow{\gamma_c}$$

The accelerations corresponding respectively to: the one in the frame of reference linked to fixed axes ($\mathcal{R}'$); that in the frame of reference linked to moving axes ($\mathcal{R}$), the inertial acceleration; finally Coriolis acceleration.

So, Ampère's article contains, in advance - but in a case very particular and unrelated to the majority of the system as studied by Coriolis - the results of the latter: those on the inertial force (1831) and those on the compound centrifugal forces (1835).

## FOUCAULT'S PENDULUM (1851) - AN APPLICATION?

In his lifetime, Coriolis had the satisfaction to see the compound centrifugal force enter manuals on *rational mechanics*. But he was undoubtedly far from imagining what was going to happen hardly 8 years after his death – an astounding experimental verification of the compound centrifugal force in a fundamental area unrelated to the theory of machines!

This is almost a textbook case in the history of science, where two *related* results are highlighted *independently* in such a short space of time (fifteen years). Especially since everything separates Foucault from Coriolis: Foucault was a physicist (*cf.* his experiments on the speed of light[9]), interested in astronomy, rather experimental, a *self-made man* (his first degree is his thesis in

8. In the formulas of type **f** = m$\gamma$, the Coriolis effect may appear either to the right, as such *acceleration of Coriolis* $2\Omega\wedge v$, either be tilted to the left with the opposite signs and appears as *Coriolis effect* $2mv\wedge\Omega$. It is also the inertial acceleration: for example, one of the two terms of *(1)*, - $r\theta'^2$**u**, can be tilted to the other side and appear in $mr\theta'^2$**u**, indicated as centrifugal inertia force.
9. See Jean-Jacques Samueli, 'L'expérience du miroir tournant de Foucault' (1853), *BibNum*, September 2009.

physics in 1853, at thirty four years old, also a scientific journalist (in the *Journal des Débats*) and a popularizer; Coriolis on the other hand is of a different generation (since twenty-seven years separate them), he was of mathematical training/education, rather theoretical, brilliant Ecole polytechnique civil engineer, with very little concern for the communication or the popularization of science[10].

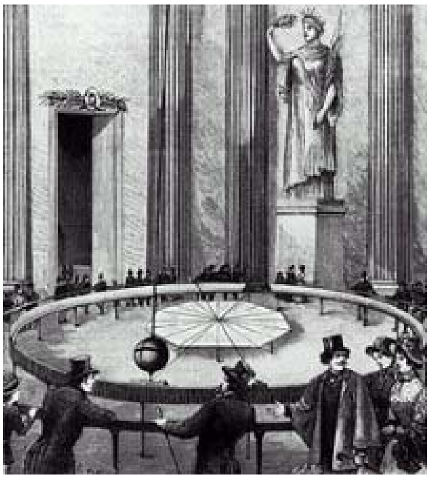


***Figure 5:* The experiment of Léon Foucault's pendulum at the Pantheon in Paris, in 1851.** This pendulum is relocated in the Pantheon in 1995 *(© Illustration Conservatoire national des arts et métiers).*

Moreover, their paths did not cross - their results come absolutely independently. Dugas (*op. cit.*) rubs it in as follows in concluding his chapter IV which is dedicated to the relative movement:

> *Compound centrifugal force within Coriolis and Foucault's pendulum are two essential conquests of mechanics, one especially of mathematical origin, the other the result of a brilliant physicist's intuition, that today the classic papers come to a single rational explanation, born separately: it is therefore not the reading of Coriolis that inspired Foucault's experiment.*

Costabel[11] also focused on this topic. Regarding the theoretical aspect, he thinks that the promoters of mechanics in the 18th century and early 19th century were 'more worried about developing all mathematical consequences of the principles posed for the dynamic analysis of the movement that to establish a reflection on the impact that could have in this analysis the paid attention to the frame of reference of the movement' – he stresses in this context the remarkable character of Coriolis's approach. Regarding the practical aspect, he indicates with reason that Foucault's pendulum owes more 'to the sharp sense of experimentation of its author rather than a clear theoretical vision of the

10. See Alexandre Moatti 'Sur le bruit du tonnerre', *BibNum* analysis of a popular Coriolis text, May 2009.
11. Pierre Costabel, Article sur la mécanique in *Histoire générale des sciences. La science contemporaine 1. Le XIX^e^ siècle*, Dir. René Taton, Quadrige, P.U.F.

problem'. And, paraphrasing Dugas in a less concise yet more academic style, he concludes, regarding these two results:

> *Born separately, the classic papers have been bringing them together since the early 20th century in a single rational explanation, but if this took a long time to develop, this is precisely because of the difficulty in bringing out from these two conquests their common and essential lesson.*

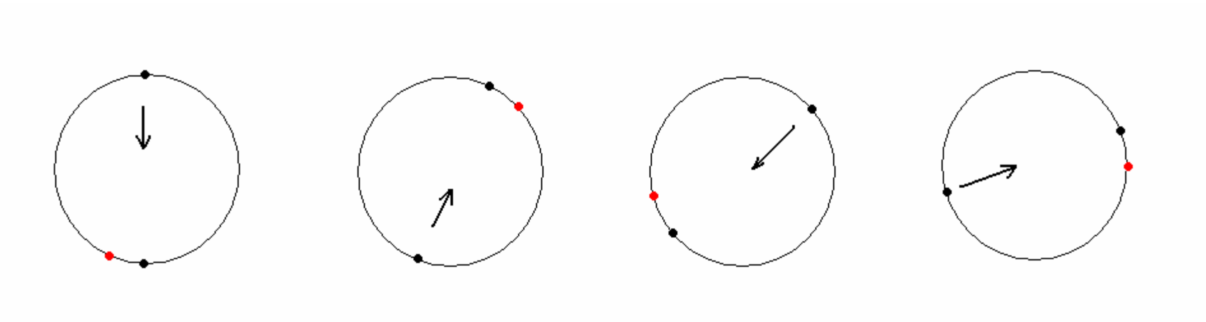


***Figure 6:* Graphic explanation of Foucault's pendulum via the Coriolis force**. *On the left, the pendulum launched at noon towards 6 o'clock is deflected to its right by the Coriolis force, it arrives at around 7 o'clock (the red dot); the following circle, the right dot at 7 o'clock is replaced with a black dot: launched from this point to 1 o'clock, the pendulum is deflected to its right by the Coriolis force and arrives at 2 o'clock (red dot), and so on. On these four diagrams, in them roundtrips of the pendulum, the plane of oscillation of this has already made a quarter turn (NB: the representation here is schematic, the rotation of the plane of the pendulum is in fact done more slowly, but follows the same diagram). As Foucault wrote [in 1851], 'it seemed to me that the mass of the pendulum could be likened to a missile that deflects towards the right when it moves away from the observer' (diagram by A. Moatti)*

### REICH AND THE DEVIATION OF THE BODIES TO THE EAST

But another experimental result, less well-known nowadays and less studied than Foucault's pendulum, had been discovered beforehand in another context. Ferdinand Reich, German chemist and physicist (1799-1882) had highlighted in 1833 the deviation of heavy bodies to the east: in a mine shaft in Freiberg (Saxony), of a depth of 158 m, he had measured on average, after 106 tests, a deviation of 28.3 mm. This deviation to the east is calculated according to the vector formula of Coriolis $2\,\mathbf{v} \wedge \Omega$ : it is worth $2/3\omega T_0 h\cos\alpha$, where $\omega$ is the speed of rotation of Earth, *h* the height of the fall, $T_0$ the time of the fall ($T_0 = \sqrt{(2h/g)}$) and $\alpha$ the latitude of the place. Poisson himself, in a paper in 1837 to Académie des sciences[12], had studied this deviation, resuming Reich's experiments – but doing so without making a link to Coriolis's work, dating back two years previous.

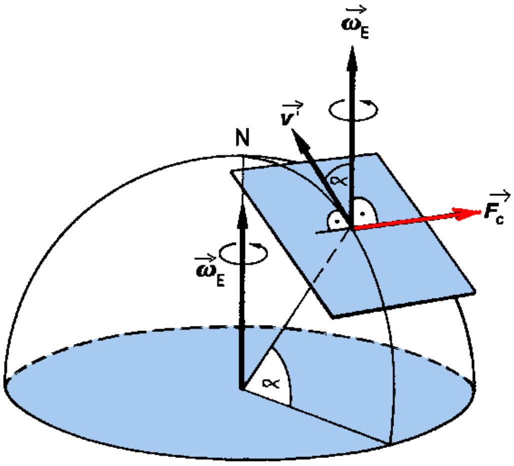



***Figure 7: Representation of the Coriolis force, directed towards the east for a located moving object approaching the axis of rotation Earth ($F_c = 2mv\omega \sin\alpha$).***
*To be as concise as possible to describe the Coriolis force (in the simplified case of a moving object on Earth's surface, there is a choice between two equivalent assertions: 1°/any moving object approaching Earth's axis of rotation undergoes a deviation to the east, while any moving object moving away undergoes is deflected westward; 2°/any moving object in the northern hemisphere is deflected to its right; while any mobile in the southern hemisphere is deflected leftward.*

Foucault knew this result through Poisson's article which mentions it. However, as he notes about this effect of deviation, 'the pendulum presents the advantage to accumulate the effects and to make them pass the field of the theory in that of the observation[13]'. They are indeed remarkable experimental conditions 'that minimize the amortisation of the slow oscillations (of the pendulum) and allow to extend for quite long enough the observation to enjoy 'the accumulation of the effects' (Costabel, *op. cit.*).

It should be noted that Foucault cites Poisson, but not Coriolis whose work he did not know, as we have said: his theoretical reference is that of Poisson's effect of deviation. Let's also note that Foucault's vision develops in areas unrelated to those of Coriolis - basic astronomy and celestial mechanics: he wants to demonstrate Earth's dirunal movement, as indicated by the title of his 1851 article in the *Comptes-rendus.*

12. 'Extrait de la première partie d'un Mémoire sur le mouvement des projectiles dans l'air, en ayant égard à leur rotation et à l'influence du mouvement diurne de la Terre', *Comptes-rendus de l'Académie des sciences*, 1837, t.5, p.660-668 (meeting, 13th Novembre 1837).
13. Léon Foucault, 'Démonstration physique du mouvement de la Terre au moyen du pendule' [Physical Demonstration of the Earth's Motion of Rotation, by Means of the Pendulum], *Comptes-rendus de l'Académie des sciences*, 1851, p.135-139.

## THE BERTRAND-BABINET-DELAUNAY DEBATE ON THE EROSION OF WATERWAYS (1859)

Scientific bodies have only gradually comes to accept the concept of Coriolis in all its generality, taking it out of its initial field of application and disregarding Coriolis's demonstrative approach. After having been shown again kinematically and extracted from the theory of machines, Coriolis's theorem will explain numerous phenomena, such as Foucault's pendulum or the erosion of waterways.

Charles-Eugène Delaunay (1816-1872), in France, would contribute the most to explaining the majority of application of Coriolis's results. In 1856, in his *Traité de mécanique rationnelle*, he writes passively and without citing Coriolis:

*The second apparent force was named compound centrifugal force.*

But the movement launched: it is in using this force that Delaunay gives theoretical basis to Foucault's experiment: four years after, this constitutes one of his first explanations in a manual.

***Figure 8:* Charles-Eugène Delaunay (1816-1872).** *Ecole polytechnique student, astronomer and mathematician.*

@@@@@@@

Some time later, in 1859, a fairly virulent debate took place between Joseph Bertrand, Babinet and Delaunay, where the latter would decisively evoke Coriolis, against Bertrand [14].

Jacques Babinet (1794-1872), himself, talks of a Foucault theorem which 'rectifies and completes several theories, accepted and professed by scholars of the first order' whereby a free point going westward with a speed acquires 'to the

14. *Comptes-Rendus de l'Académie des sciences*, meeting on 14th Novembre 1859, p. 685-693 'Seconde note sur l'influence du mouvement de la Terre' [Second note on the influence of the motion of the Earth], by M. BERTRAND; 'Sur le déplacement vers le nord ou vers le sud d'un mobile qui se meut librement dans une direction perpendiculaire au méridien' [[On the moving towards the north or the south of a mobile that moves freely in a direction perpendicular to the meridian]], by M. BABINET; Observations of M. DELAUNAY on the same issue; Response of M. BERTRAND to M. DELAUNAY; then M. PIOBERT.

north, i.e., to the right', a relative speed equal to $\omega a \sin\lambda$. But it is especially Delaunay who opens, in Babinet's defence, a debate with Bertrand on the erosion of waterways - which is an application of the Coriolis force: Bertrand thought that only waterways directed along the meridians eroded their right bank (and not the waterways directed from the east to the west); Delaunay shows him otherwise.

To do this, Delaunay explicitly makes reference to Coriolis, while focusing on the simplification made of his approach:

> *The study of these relative movements, the search of these particularities that they present and that can be distinguished from absolute movements, is extremely delicate. The step that seems to me the most suitable to get there, consists of relying on a strongly ingenious theory that we owe to Coriolis, and that has been so simplified in recent years, that it has been incorporated into the ordinary teaching of rational mechanics: I want to talk about the* theory of the apparent forces in the relative movements.

Delaunay grants Coriolis due to his naming of the *compound centrifugal force*, that he 'completely determined the value, direction and sense', and that it

> *(…) leads to the rotation of the plane of oscillation of the pendulum, in the experiment of M. Foucault; this is what produces the movements observed in the gyroscope of the same physicist; it is finally this that intervenes in the movement of waterways, and that which tends to bring the water to the right side of their bed.*

He gives the simplified expression of the compound centrifugal force $2m\omega a \sin\alpha$. The debate continues between Delaunay and Bertrand. The latter confirms his reluctance in using 'Coriolis's compound centrifugal forces': it is precisely because they are 'fictitious' that they 'do not seem likely to understand the mechanism of the phenomenon', which Bertrand does not seem to understand very well himself, since he gets a wrong result on the waterways east to west: perhaps this is, also, for Bertrand, a way to reject his own fault onto Coriolis? Let's also leave the word conclusive, particularly relevant to Delaunay:

> *M. Bertrand (…) seems to be reluctant to use Coriolis's fictitious forces to come to the explanation of the real phenomena that show us the existence of the rotation of earth. I do not claim to say that the theory of Coriolis alone can demonstrate it. But I have just seen that this theory leads very easily to a clear and precise idea of the way things should happen. I add that in any way that we reason, following another course, it is necessary that we arrive identically at the same results (…)*

Finally, it seems that this debate of 1859 at Académie des sciences, with this assumption of Delaunay, note the fact that it is worth assigning Coriolis the idea and the theorization of the compound centrifugal forces[15]. The actual arguments providing a damper on Coriolis's theory (approach by the too complicated dynamics, possibility to make calculations in the inertial frame of reference without 'fictitious' forces) have certainly been recognised, but the learned body assigned him the original idea and cites him. From this date, all applications pertaining to the theory of Coriolis (Foucault's pendulum, erosion of waterways) is clearly linked, even if we speak not of the *Coriolis force* but *compound centrifugal forces*. For example, in 1863, in a paper quite education at the Academy[16], Finck explains a passage of Arago's *l'Astronomie* on the deviation of bodies while using 'Coriolis's theory of relative movements', and while using his equations: the bend is made to use Coriolis's results in areas that are not his.

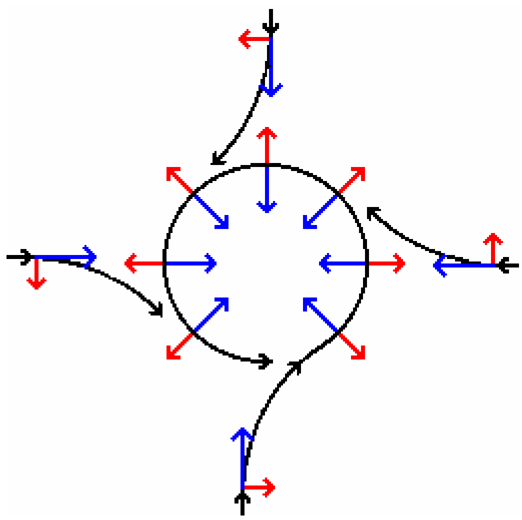
This force enters gradually, also, in the most academic papers of rational mechanics. By way of example, some decades later, Paul Appell, in his *Traité de mécanique rationnelle* of 1896, mentions the 'compound centrifugal force'. In 1930, the class of Paul Painlevé at l'École polytechnique mentions the 'compound centrifugal force' ten times, while even indicating during one of the occurrences 'compound centrifugal force or Coriolis effect'.

## THE CORIOLIS EFFECT IN METEOROLOGY

Anders Persson specialised in the history of the Coriolis force in this area, on its meanings and how to present it. He connects it in an interesting way to the simple centrifugal force, and is also quite radical on how to explain:

> *In contrast to the 'normal' inertia, that resists changes of movement of a body, the inertial force of Coriolis resists these displacements while trying, by circular motion, to bring back the body to its original position.* Any mathematical or intuitive explanation of the Coriolis force that would enter in conflict with the notion of circular inertial motion would therefore be incomplete or false.

15. This is, for example, the opinion of Anders Persson, 'How do we understand the Coriolis force?', *Bulletin of the American Meteorological Society*, 79, n°7; Jully 1998 (PDF Princeton University)
16. Finck, 'Chutes des corps qui tombent d'une grande hauteur' [Drops of Bodies that Fall From a Great Height] *CRAS*, 1863 (T.56), p. 957 et ss.



***<u>Figure 9:</u>* formation (anticlockwise in the northern hemisphere) of a cyclone around a depression.** *The blue arrows indicate the winds' force of attraction, always towards the centre of the depression (high to low pressure); the red arrows indicate the Coriolis deviation, always oriented towards the right course. When the air mass approaches (here a quarter of the four sides), it is deflected towards its right; it is rotated, undergoing still the Coriolis force (this time centrifugal compared to the centre of the depression) opposed to the blue force, centripetal. The air masses thus twist and turn instead of going in a straight line towards the centre of the depression.*

The application of Coriolis force in the field of meteorology will not come from France: we have seen that our French physicists, from Foucault to Delaunay, have already greatly expanded the original area of application of the compound centrifugal force, and have gave it a legitimacy in relation to Coriolis.

Persson dates from 1858 (at about the same era as the French debates between Bertrand and Delaunay) the introduction of a rotary centrifugal force in meteorology, spurred on by William Ferrel (1817-1891):

> *If a body is moving in any direction, there is a force arising from the earth's rotation, which always deflects it to the right in the northern hemisphere, and to the left on the southern.*

Persson's approach is interesting. Against the majority of scientists, he thinks that Coriolis's approach by dynamics is, in any case for meteorology, a lot more productive than the kinematic approach – he regrets that Coriolis's work could have been truly known only with the Jacques Gabay 1990 reissue (!). According to him, if the major discoveries in this field were made 'without particular knowledge' of Coriolis's works, their dissemination had avoided in his opinion many incorrect meteorological interpretations - in conclusion he also pays tribute to Coriolis:

*That is why he is absolutely qualified to lend thus his name to the eponymous force. Had he been with us today, without doubt he had would have been probably one of the few who understood it and taught correctly!*

We will not go further into the history of the Coriolis force in meteorology - particularly as it is an area that was without a doubt even more foreign to the concerns of Coriolis than the celestial mechanics of Foucault. In the same way that we could locate and date in France the granting of Coriolis's works of the effects of the 'compound centrifugal force' (the use of this naming being as a clear reference to the 1835 works of Coriolis), it would be interesting to locate the date this concept of the compound centrifugal force, or Coriolis force, started to spread outside of France, in the field of meteorology namely. However, this goes well beyond the scope of this article.

■

*(October 2011)*

*(translated in English by John Moran, published June 2014)*