

On the Josephus problem

by Laurent Signac
Researcher, Laboratory for Systems Informatics and Automatics
(LIAS, EA 1219, University of Poitiers)
Lecturer in Computer Science,
École Nationale Supérieure d'Ingénieurs de Poitiers (ENSIP)

The Josephus problem is a highly addictive conundrum if ever there was one. A relatively well-known arithmetical puzzle that crops up regularly in popular mathematics and even computer science, it is part of the family of *decimation* problems. The somewhat bloody origin¹ of this word makes perfect sense in the context of the Josephus problem.

The problem statement is very simple: 41 people are standing in a circle and a starting point is chosen (Figure 1a).

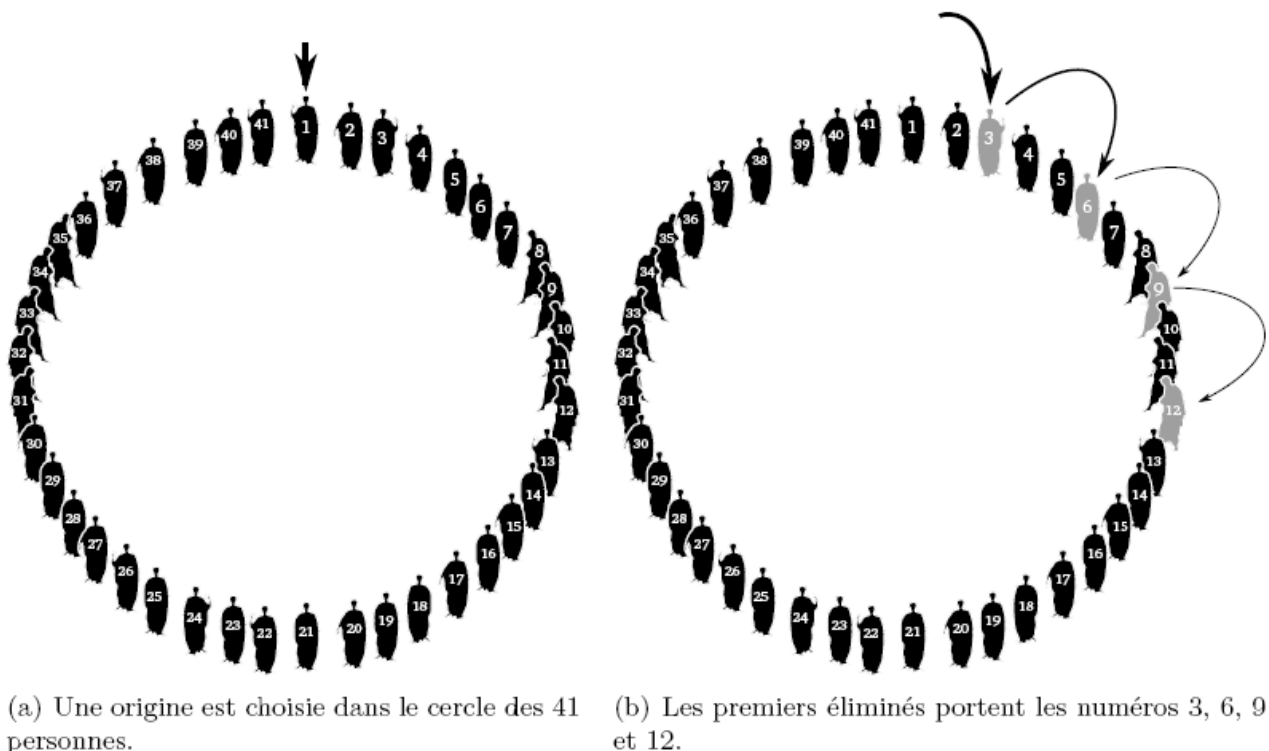


Figure 1: The decimation process.

1. The term *decimation* derives from the Roman military punishment of “eliminating” one out of ten people in a group. The person was probably chosen by drawing lots.

Starting with this person, we count 1, 2, and the third person is removed.² The process continues: 1, 2, and the third person is removed. The first people to leave the circle are therefore numbered 3, 6, 9 and 12, as shown in Figure 1b. The same process is repeated around the circle (Figure 2a). Notice that after one full circle, empty places appear, which complicates matters. Here are the numbers of the unlucky losers, in the order of their elimination (Figure 2b):

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 1, 5, 10, 14, 19, 23, 28...

The question is: Where to stand in the circle to be last survivor? In other words, what will be last remaining number in the circle?

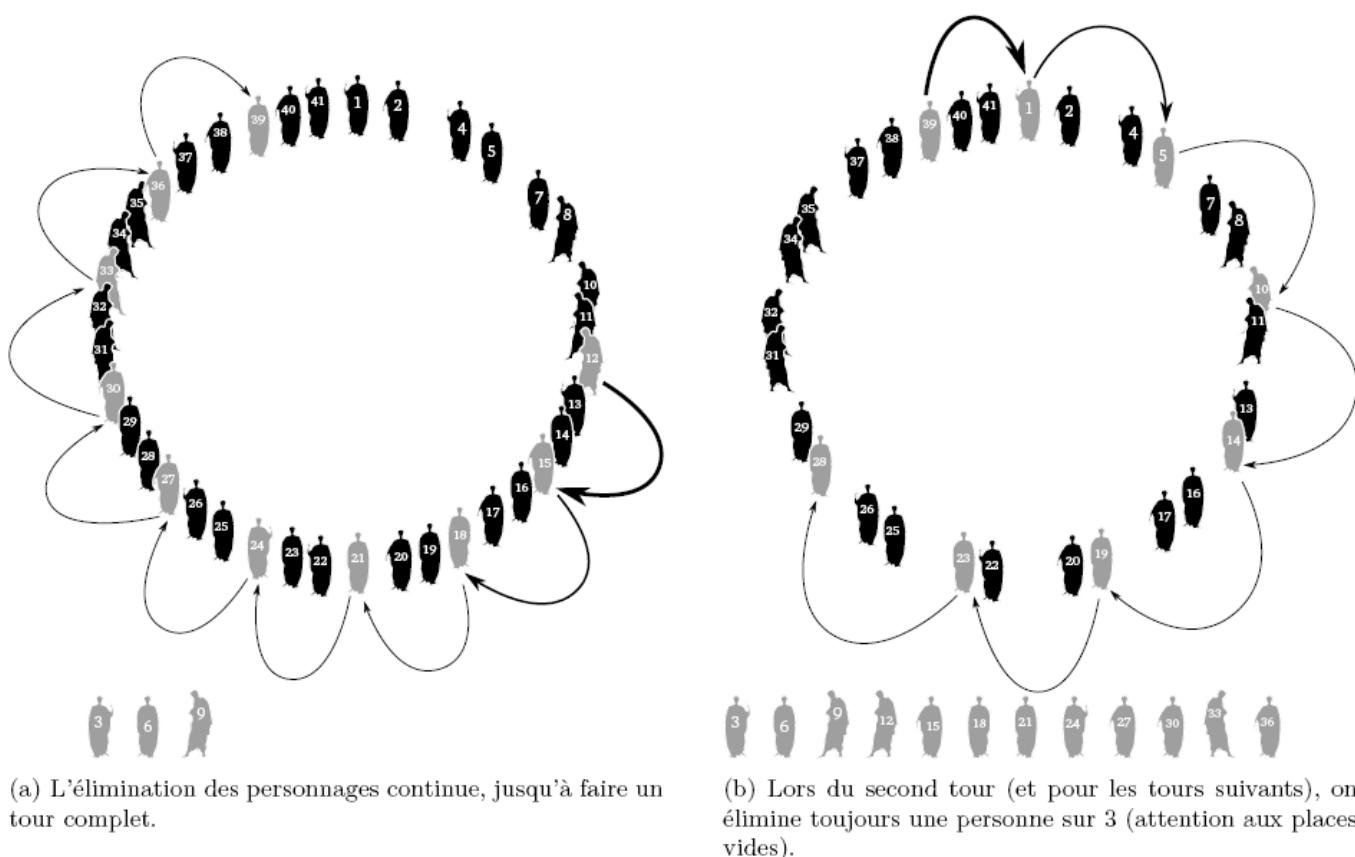


Figure 2: The decimation process continues.

2. The term *decimation* will be used here even if a person is selected from a group of n people, where n is other than 10.

I – THE JEWISH WAR

The above problem is generally connected to Flavius Josephus, a “historian” of the first century AD, who related this episode of his life (he was one of the 41 men) in his work *The Jewish War*. Each person singled out in the process described above was killed by the next person selected. Clearly, Flavius Josephus knew how to solve the problem.

Mathematically speaking, in the standard scenario (for a given number of people, not necessarily counted in threes) the solution cannot be obtained by direct calculation.³ Indeed it is probably the absence of a direct and general solution that accounts for the problem’s enduring appeal and makes it intriguing even today. What’s more, regardless of the initial input data, it is relatively straightforward to *simulate* the decimation phase using a computer to obtain the number of the last survivor in the circle and the order of elimination: such a programme is fun to write and budding programmers are encouraged not to read the suggested solution at the end of this article before working it out for themselves.

The historical dimension of the story can only pique curiosity and it is difficult to resist the temptation to seek further details on how the original hero saved his skin in those distant times. Contemporary accounts describe how, during the Siege of Jotapata by Vespasian⁴ in 67 AD, 41 men (Flavius Josephus, then governor of the city, and 40 soldiers) were cornered in a cave, refusing to surrender. This historical context allows us to gauge Flavius Josephus’s prowess: the situation in which he found himself, holed up in a cave knowing that the slightest error would cost him his life, is hardly conducive to the serenity of mind required to solve a cerebral problem involving a circle of 41 men.

The temptation is therefore great to seek out further information, though not in book of mathematical puzzles but in one of the many translations based to a greater or lesser extent on Flavius Josephus’s text. The first such publication is in French and available on Gallica: it is the *Œuvres complètes de Flavius Josèphe*, edited by Théodore Reinach, translated from the Greek by René Harmand, and published from 1911 onwards. The extract is found in Tome III, chapter VIII, of *The Jewish War*:⁵

3. Some solutions are based on recurrences, others on the calculation of annex sequences (“rounded” geometric sequences), but there is no direct calculation in the standard scenario.

4. Who would become emperor two years later.

5. Source [Gallica](#).

388. met alors sa confiance dans la protection de Dieu : « Puisque, dit-il, nous sommes résolus à mourir, remettons-nous en au sort pour décider l'ordre où nous devons nous entretuer : le premier que le hasard désignera tombera sous le coup du suivant et ainsi le sort marquera successivement les victimes et les meurtriers, nous dispensant d'attenter à notre vie de nos propres mains. Car il serait injuste qu'après que les autres se seraient tués il y en eût quelqu'un qui pût changer de sentiment et vouloir survivre ¹. » Ces paroles inspirent confiance, et après avoir décidé ses compagnons, il tire au sort avec eux. Chaque homme désigné présente sans hésitation la gorge à son voisin dans la pensée que le tour du chef viendra bientôt aussi, car ils préféreraient à la vie l'idée de partager avec lui la mort. A la fin, soit que le hasard, soit que la Providence divine l'ait ainsi voulu, Josèphe resta seul avec un autre : alors, également peu soucieux de soumettre sa vie au verdict du sort ou, s'il restait le dernier, de souiller sa main du sang d'un compatriote, il sut persuader à cet homme d'accepter lui aussi la vie sauve sous la foi du serment ².
392. 8. Ayant ainsi échappé aux coups des Romains et à ceux de ses propres concitoyens, Josèphe fut conduit par Nicanor auprès de Vespasien.
393. De toutes parts les Romains accouraient pour le contempler et, autour du prétoire ³, il y eut une presse énorme et un tumult en sens divers : les uns se félicitaient de la capture du chef, d'autres proféraient des menaces, quelques-uns se poussaient simplement pour le voir de plus
394. près. Les spectateurs les plus éloignés criaient qu'il fallait châtier cet ennemi de Rome, mais ceux qui étaient à côté se rappelaient ses belles actions et ne laissaient pas d'être émus par un si grand changement. Parmi les généraux il n'y en eut pas un qui, si fort qu'il eût
395. d'abord été irrité contre lui, ne se sentit quelque pitié à sa vue : Titus
- 396.

1. Destinon supprime cette phrase comme interpolée. D'ailleurs, même avec le procédé suggéré par Josèphe, il y aura toujours un dernier survivant qui devra forcément se tuer lui-même.

2. Il est permis d'avoir des doutes sur l'authenticité de cette historiette qui a fourni aux mathématiciens, depuis la Renaissance, un piquant sujet de problème.

3. Nous lisons avec Holwerda : περί τὸ στρατήγιον (mss. : τῷ στρατηγῷ, τοῦ στρατηγοῦ, etc.).

Here is the passage in question:

Since, he said, we are resolved to die, let us leave it to fate to decide the order in which we shall kill one another: the first man designated by chance will fall by the sword of the next such man, and thus fate will successively mark out the victims and their killers, sparing us from taking our lives with our own hands [...] Finally, by luck or because Providence had ordained it, Josephus alone remained with one other man: then, loth to submit his life to fate or, if he alone remained, to sully his hands with

the blood of a compatriot, he persuaded the man, under oath, to spare their lives.

Algorithmically speaking, this is hardly satisfying. This, it seems, is more a tale of chance and divine Providence than prodigious calculation. Apart from the number of men, which is indeed 41 in this text (the figure is referred in the previous pages), there is no mention of a circle, nor of any algorithmic means of selecting the men to exclude from the circle by counting in threes.

In a footnote, the translator René Harmand voices some reservations about the authenticity of the story told by Flavius Josephus, while nevertheless admitting that since the Renaissance it has given rise to a “piquant mathematical problem”.

Finally, the text hints that when only Josephus and his companion remain alive, the former suggests putting an end to the decimation process for fear of dying or being forced to kill. Yet, at the moment, Josephus necessarily knew in which of the two possible situations he found himself. If he committed the final murder (which is unlikely, since he is loth to become a murderer), it is his turn to die. If on the other hand, it was committed by his companion, it is Josephus’s turn to become the executioner and his survival is therefore assured. As he most probably found himself in the second situation, it follows that Josephus really did spare his companion, probably so as not to become an executioner himself. This imprecision further undermines the story’s credibility.

However, the translation used in the present article was of course based on texts other than the originals⁶ (which, it seems, are no longer extant). It is quite conceivable that several versions of this passage exist, and that these vary in how much detail they give on Flavius’s method and ingenuity. Below, for example, is the same passage taken from the fifth volume of *The Jewish War* (Tome III, Chapter XLIV). This translation from the Greek by Arnauld d’Andilly has been republished many times since the 17th century (below, the edition from 1744):

6. It is thought that Flavius Josephus wrote in Aramaic and then produced his own translation into Greek. The Latin translations appeared later.

tés, ils ne pouvoient s'empêcher de rêver un chef pour qui ils avoient tant d'estime, ils sentirent leurs bras s'affoiblir; leurs épées leur tomboient des mains, & dans le même-tems qu'ils lui portoient quelques coups, leur affection pour lui s'opposant à leur colere en diminuoit tellement la force, qu'elle les rendoit inutiles.

Joseph de son côté ne perdoit point le jugement dans un si pressant péril: mais se confiant en l'assistance de Dieu, il leur parla en ces termes: « Puisque vous êtes résolus de mourir, jettons le sort pour voir qui sera celui qui devra être tué le premier par celui qui le suivra: & continuons toujours d'en user de la même sorte, afin que nul de nous ne se tuë de sa propre main, mais reçoive la mort par celle d'un autre. » Cette proposition fut reçue de tous avec joye, parce qu'ils ne pouvoient douter que Joseph ne fût bien-tôt du nombre de ceux qui seroient tués, & qui préféreroient à la vie une mort qui leur seroit commune avec lui.

270. Ainsi le sort fut jetté: & celui sur qui il tomboit, tendoit la gorge à celui qui le devoit tuer: ce qui continua jusques à ce qu'il ne resta plus que Joseph & un autre, soit que cela arrivât par hazard, ou par une conduite particuliere de Dieu. Alors Joseph voyant que s'il eût encore jetté le sort, ou il lui auroit fallu tremper ses mains dans le sang d'un de ses amis, il lui persuada de vivre, après lui avoir donné parole de le sauver.

The text reads:

As you are resolved to die, let us cast lots to see who will be killed first by the man who follows: & let us continue in the same manner [...] Thus the lots were cast: & the man on whom it fell leant his throat towards the man who was to kill him: this continued until only Josephus & another man were left, whether by chance or an act of God. Then Josephus, seeing that

if he threw lots again, it would either cost him his life, or he would have to soak his hands in the blood of one of his friends, persuaded the other to live, after giving his word to save him.

While it seems clearer here that Josephus spared the second-to-last survivor, we are still left wanting to know more about the actual decimation process that was used: there is no circle, fate picks the victims, and Josephus gets out alive without calculating a thing.

@@@@@@

Lastly, there is a third version, known as the *Capture of Jerusalem*, written in Slavic (Old Russian) and based on various manuscripts from the 15th and 16th centuries containing the entire text or fragments of it mingled with other chronicles. These manuscripts may be derived from older Slavic translations (12th or 13th centuries) of even earlier and no longer existent texts. This Paleo-Slavic version differs in several ways from the European versions with which we are familiar. In particular, it contains details about the lives of Jesus and John the Baptist. We can only speculate whether these details were added by the Slavic translator or whether he translated from a lost version which contained such details. The answer may be a bit of both.

A French translation of the Old Russian text by Pierre Pascal was published in two volumes by the Institut d'Études Slaves in 1934 and 1938, under the title *The Capture of Jerusalem by Josephus the Jew*. The Russian text was also reprinted to accompany the French text. In 1965, the same translation, minus the text in Russian, was published by the Éditions du Rocher.

The Paleo-Slavic version contains a description of the episode in the cave. Here is P. Pascal's translation:

Thus Josephus, entrusting his salvation to God, said: "Since God wishes for our deaths, let us die by counting: the man at the end of the count will be executed by his counterpart in the next count." Having spoken thus, he counted so shrewdly that he outwitted all. All except one man were killed by the other men. Josephus, anxious not to taint his hands with the blood of a compatriot, pleaded with him], and both men walked out alive.

This version is noticeably different from those we have considered up to this point. While this is still a matter of *entrusting one's salvation to God*, the expression *he counted so shrewdly that he outwitted all* leaves little room for ambiguity.

Below is a handwritten copy of the Slavonic text:

И ТАКО РЕКЪ, ПОЧТЕ ЧИСЛА СЪ МОУДРОСТНО,
И ТЪМ ПРИБЛАДЕ ВСА.

The term translated as *shrewdly* (МОУДРОСТНО) can also be interpreted as *wisely*, which confers a different meaning (almost one of resignation) to the sentence.

Traces of Josephus the number cruncher are not non-existent, but there is a lack of technical details about the decimation process. What's more, scope for further investigating this text is limited. Thus, rather than seeking new translations of Josephus's tale, we can turn the problem on its head and try to find out where the authors of mathematical puzzles unearthed this idea for an arithmetical puzzle. This will perhaps lead us to one particular version of *The Jewish War*.

II – ARITHMETICAL AMUSEMENTS

The tale is now so widely known that recent publications describe the *famous episode in the cave* without giving an exact source. One has to turn to specialist authors of the 19th and 20th centuries to obtain new information. In particular, identical or similar problems are found (for example) in publications by Édouard Lucas, an author well known for inventing the – for IT students – endlessly maddening Tower of Hanoi puzzles, and Ernest Dudeney.

Ernest Dudeney's book *Amusements in Mathematics*, published in 1917, presents readers with a puzzle similar to the Josephus problem but involving cats and mice. It does not really advance our quest to discover the truth about Flavius's calculatory prowess, but does provide an entertaining variation on the theme.

Dudeney's work is available on Project Gutenberg.⁷ Puzzle 232 is reproduced below along with the illustration that accompanied the puzzle in the original edition.

7. [Page](#) on Gutenberg.



"Play fair!" said the mice. "You know the rules of the game."

"Yes, I know the rules," said the cat. "I've got to go round and round the circle, in the direction that you are looking, and eat every thirteenth mouse, but I must keep the white mouse for a tit-bit at the finish. Thirteen is an unlucky number, but I will do my best to oblige you."

"Hurry up, then!" shouted the mice.

"Give a fellow time to think," said the cat. "I don't know which of you to start at. I must figure it out."

While the cat was working out the puzzle he fell asleep, and, the spell being thus broken, the mice returned home in safety. At which mouse should the cat have started the count in order that the white mouse should be the last eaten?

When the reader has solved that little puzzle, here is a second one for him. What is the smallest number that the cat can count round and round the circle, if he must start at the white mouse (calling that "one" in the count) and still eat the white mouse last of all?

And as a third puzzle try to discover what is the smallest number that the cat can count round and round if she must start at the white mouse (calling that "one") and make the white mouse the third eaten.

Clearly it is the same problem. While it appears to have been turned on its head, Josephus's solution (Who is last?) and the cat's solution (Where to start to finish at a predetermined position?) are interchangeable.⁸ The cat-and-mouse problem is amusing, but unfortunately it does not bring us any closer to the origin of the Josephus problem. For his part, Édouard Lucas proffers his version

8. We move from one problem to the other by changing the start point of the numbering, in other words by shifting the numbers.

in *Arithmétique amusante*, published posthumously in 1895.⁹ After expounding another variant, which we will discuss later on, he gives some clues as to the origin of the Josephus problem:

le dix et la dame de trèfle, et ainsi de suite, jusqu'à ce qu'on ait retiré les quinze cartes noires.

Au lieu des deux vers français qui nous permettent de résoudre immédiatement le problème proposé, on se servait autrefois de ce vers latin, où l'on trouve les mêmes voyelles dans le même ordre;

Populeam virginem mater Regina ferebat.

Nous allons indiquer maintenant la méthode qui permet de retrouver, indépendamment de tout procédé mnémotechnique, la disposition des cartes rouges et des cartes noires. Plaçons en cercle trente cartes retournées, en indiquant la place de la première, qui correspondait au roi de carreau. En comptant de neuf en neuf, on tombe sur les cartes dont les rangs sont 9, 18, 27.

Qu'on retire ces cartes, puis que l'on continue à compter de neuf en neuf, en retirant quinze cartes, il suffira ensuite de remplacer les cartes restantes par quinze cartes rouges, en conservant la place de la première, le roi de carreau; puis on mettra à la place des cartes manquantes quinze cartes noires.

BACHET, dans la Préface au lecteur de la seconde édition de ses *Problèmes plaisants et délectables qui se font par les nombres*, nous donne l'origine de ce curieux problème :



HISTOIRES DE BRIGANDS.

« Hégésippus, au troisième Livre de la *Guerre de Jerusalem*, rapporte la mémorable histoire de Josèphe, ce fameux auteur qui nous a laissé par écrit la même *Guerre des Juifs*, lequel était

gouverneur de la ville de Jotapata, lorsqu'elle fut assiégée et peu après emportée d'assaut par Vespasien. Il fut contraint de se retirer dans une citerne, suivi d'une troupe de soldats, pour éviter la première fureur des armes victorieuses des Romains; mais il courut plus de fortune de perdre la vie parmi les siens que parmi les ennemis: car, comme il eut arrêté de s'aller rendre à la merci du vainqueur, ne pouvant imaginer aucun autre moyen de se garantir de la mort, il trouva ses soldats saisis d'une telle frénésie qu'ils voulaient tous mourir et s'entre-tuer les uns les autres plutôt que de prendre ce parti. Josèphe s'efforça bien de les détourner d'une si malheureuse entreprise, mais ce fut en vain; car, rejetant tout ce qu'il put leur alléguer au contraire, et persistant en leur opinion, ils en vinrent jusque-là que de le menacer, s'il ne s'y portait volontairement, de l'y contraindre par la force, et de commencer par lui-même l'exécution de leur tragique dessein. Alors, sans doute, c'était fait de sa vie s'il n'eût eu l'esprit de se défaire de ces hommes furieux par l'artifice du présent problème. Car, feignant d'adhérer à leur volonté, il se conserva l'autorité qu'il avait sur eux, et par ce moyen, leur persuada facilement que, pour éviter le désordre et la confusion qui pourraient survenir en tel acte, s'ils s'entre-tuaient à la foule, il valait mieux se ranger par ordre en quelque façon, et commençant à compter par un bout, massacrer toujours le tantième, jusqu'à ce qu'il n'en demeurât qu'un seul, lequel serait obligé de se tuer soi-même. Tous étant de cet accord, Josèphe les disposa de sorte, et choisit pour lui une si bonne place, que la tuerie étant continuée jusqu'à la fin, il se trouva seul en vie, ou peut-être encore, qu'il sauva quelques-uns de ses plus affidés, et de ceux desquels il se pouvait promettre une entière et parfaite obéissance. »

We read that:

Bachet, in his preface to the second edition of his Problèmes plaisants et délectables qui se font par les nombres, recounts the origin of this curious problem:

"Hegesippus, in the third tome of the War of Jerusalem, tells the memorable story of Josephus, [...] Josephus arranged the men in such a way, and chose for himself such a fine position, that when the blood-bath had come to an end, he alone remained alive [...]"

9. Available on the site archive.org.

Not only does Lucas seem to confirm that Josephus's survival was not down to luck, we now have two additional bibliographic sources:

- Claude Gaspard Bachet, seigneur of Méziriac, a 17th-century author often considered the first French author of mathematical puzzles,
- and more importantly, another vision of *The Jewish War* entitled *War of Jerusalem*, written by a certain Hegesippus, in which we find the phrase *Josephus arranged the men in such a way, and chose for himself such a fine position...*

@@@@@@

Let's first consider Bachet's text. The second edition of *Problèmes plaisants et délectables*, dating from 1624 (the first edition had been published 12 years before), is available on Gallica:¹⁰

PREFACE
fance. En outre ie ne crois pas, que ceux qui auront penetré dans ce liure plus auant eue l'escorce, le iugent de si peu de valeur, que feront ceux-là qui n'en auront leu que le titre car encor que ce ne soyent que des ieux, dont le but principal est de donner vne honnelle recreation, & d'entretenir avec leur gentillesse vne compagnie, si est-ce qu'il faut bien de la subtilité d'esprit, pour les practiquer parfaitement, & faut estre plus que médiocrement expert en la science des nombres, pour bien entendre les demonstrations, & pour se sçauoir ayder de plusieurs belles inuentions, que j'ay adioutees. Finalement, pour preuuer encore que ce liure n'est point du tout inutile, & que la cognoissance de ces Problemes peut seruir grandement en quelque occasion, ie ne veux employer que le témoignage d'Hegesippus au troisieme liure de la prise de Hierusalem. La

AV LECTEUR
il rapporte la memorable histoire de Iosephe, ce fameux Autheur qui nous a laissé par escrit la mesme guerre des Juifs, lequel estât gouverneur dans la ville de Iorapata, lors qu'elle fut assiégée, & peu apres emportée d'assaut par Vespasian, il fut contraint de se retirer dans vne cistern, suivi d'une troupe de soldats, pour eiter la premiere fureur des armes victorieuses des Romains. Mais il courut plus de fortune de perdre la vie parmy les siens, que parmy les ennemis: Car comme il eut arresté de s'aller rendre à la mercy du vainqueur, ne pouuant imaginer aucun autre moyen de se garantir de la mort, il treuva ses soldats faillis d'une telle frenesie, qu'ils vouloient tous mourir, & s'entreuer les vns les autres, plustost que de prendre ce party. Iosephe s'efforça bien de les destourner d'une si malheureuse entreprise, mais ce fut en vain; car rejetta tout ce qu'il pueit leur al-

PREFACE
leger au contraire, & persistans en leur opinion, ils en vindrent iusques là, que de le menacer, s'il ne s'y portoit volontairement, de l'y contraindre par force, & de commencer par luy mesme l'exécution de leur tragique dessein. Alors sans doute c'estoit fait de sa vie, s'il n'eust eu l'esprit de se defaire de ces hommes furieux, par l'artifice de mon 13. Probleme. Car feignant d'adherer à leur volonté, il se confesua l'autorité qu'il auoit sur eux, & par ce moyen leur persuada facilement, que pour eiter le desordre & la confusion, qui pourroient suruenir en tel acte, s'ils s'entreuoient à la foule, il valoit mieux se ranger par ordre en quelque façon, & commençant à conter par vn bout, massacrer tousiours le tantiesme. (l'Auther n'exprime par le quantiesme) iusques à ce qu'il n'en demeurast qu'un seul, lequel seroit obligé de se tuer soy-mesme. Tous estans de cet

AV LECTEUR.
accord, Iosephe les disposa de sorte, & choisit pour luy vne si bonne place, que la tuerie estant continuée iusques à la fin, il se trouua seul en vie, ou peut estre encore qu'il sauua quelques vns de ses plus affidez: & de ceux desquels il se pouuoit promettre vne entiere & parfaite obeissance. Voila vne histoire bien remarquable, & qui nous apprend assez, qu'on ne doit point mespriser ces petites subtilitez, qui aiguissent l'esprit, habilitent l'homme à des plus grandes choses, & apportent quelquesfois vne vtilité non preueue.
Reste que l'aduertisse le Lecteur, que ceste seconde edition est beaucoup plus accomplie, que la premiere: car outre qu'elle est plus correcte, elle est augmentee de plusieurs Problemes, & de la demonstration parfaite du Probleme, qui estoit le cinquiesme en la premiere edition, & qui est le sixiesme en ceste-

In this preface we read:

Finally, to reiterate that this book is far from useless, & that knowledge of these Problems can be of great use on occasion, I wish to refer to Hegesippus's testimony alone, in the third tome of the Capture of Jerusalem. There he recounts the memorable tale of Josephus, that famous author who chronicled the very same war of the Jews [...] he was forced to withdraw into a cistern¹¹ [...] and would no doubt have paid with

10. [Page](#) on Gallica.

11. Here the cave has been replaced by a cistern. This is not very surprising, and the Reverend Calmet's historical dictionary of the Bible tells us that the terms *lake* (which we will come to later), *cistern*, *cave* and even *tomb* could all refer to water reserves or sepulchres.

his life had he not the wit to rid himself of those raving men, by availing himself of the ploy that is my Problem 23. Pretending to be of their mind [...] [he] readily persuaded [them] [...] [that] it were better to stand in an order of a sort, and by counting from one end, always execute the nth man [tantième] (the author does not specify which [quantième]) until there would remain one man alone, who would be obliged to end his own life. [...] Josephus arranged the men in such a way, and chose for himself such a fine position [...] that he alone remained alive.

The Problem 23 in question, a variant involving Turks and Christians on a boat, was the one that inspired Édouard Lucas. Here is the text, taken from the same edition from 1624:

174

**PROBLEME
XXIII.**

Estant proposé quelque nombre d'unités distinguées entre elles, les disposer & ranger par ordre en telle sorte, que reietant tousiours la neufiesme, ou la dixiesme, ou la tantiesme que l'on voudra, iusques à un certain nombre, les restantes soyent celles que l'on voudra.

N à accoustumé de proposer ce Probleme en ceste sorte, Quinze Chrestiens & quinze Turcs se treuuent sur mer dans vn mesme nauire, & s'estant esleuee vne terrible tourmente, le pilote dit qu'il est necessaire de jeter dans la mer la moitié des personnes qui sont en la nef, pour sauuer le reste. Or cela ne se peut faire que par sort; Partant on est d'accord que se rangeans tous par ordre, & contant de neuf, en neuf, on jette chascune neufiesme dans la mer iusques à ce que de 30. qu'ils sont, il n'en demeure que 15. On demande comment il les faudroit disposer pour faire que le sort tombat sur les 15. Turcs sans perdre aucun des Chrestiens. Pour faire cecy

qui se font par les nombres. 175

cecy promptement, remarque ces deux vers:
*Mort tu ne falliras pas
En me livrant le trépas.*

Et pren garde seulement aux voyelles a e i o u. T'imaginant que la premiere a, vaut vn, la seconde e, vaut 2. la troisieme i, vaut 3. la quatrieme o, vaut quatre, & la cinquiesme u, vaut 5, & d'autant qu'il faut commencer par les Chrestiens, en la premiere syllabe (Mort) la voyelle o te monstre qu'il faut en premier lieu mettre 4. Chrestiens; en la seconde syllabe (Tu) la voyelle u te monstre qu'il faut apres ranger 5. Turcs. Ainsi (ne) signifie 1. Chrestiens; (fal) vn Turc; (li) 3. Chrestiens; (ras) vn Turc (pas) vn Chrestien; (en) 2. Turcs; (me) 2 Chrestiens; (li) 3. Turcs; (vrant) vn Chrestien; (le) 2 Turcs; (tres) 2. Chrestiens; (pas) vn Turc. La regle generale pour faire le mesme en tout nombre depend de ce que ie diray en la demonstration.

DEMONSTRATION.

Voulant faire ce ieu en quel nombre que sois, par exemple en 30. imagine toy 30. unités toutes semblables comme celles que tu vois icy descrites, & commençant à conter par la premiere, marque la neufiesme ou la tantiesme que l'on voudra avec quelque signe comme mettant dessus quelque marque, puis conte despuis celle que tu as marquée, de la mesme facon, & marque aussi la neufiesme, & continue à faire le mesme recommençant quand tu seras au bout, & sautât toutes celles que

The aim of the ship's captain is to arrange 30 people (15 Christians and 15 Turks) in such a way that by removing 15 of them and counting in nines, the 15 survivors would be Christians. Bachet provides a mnemonic to remember the positions to be assigned to the Turks and Christians:

*Mort tu ne falliras pas
En me livrant le trépas*

[Death you will not fail
To bring about my demise]

Each vowel is assigned a number (1 for *a*, 2 for *e*, 3 for *i*, 4 for *o* and 5 for *u*) and the couplet therefore equates to a series of numbers: 4, 5, 2, 1, 3, 1, 1, 2, 2, 3, 1, 2, 2, 1, indicating that the arrangement is 4 Christians, 5 Turks, 2 Christians, etc.

The final arrangement is given in Figure 3. Many other mnemonics exist in various languages (Lucas gives one in Latin, which we will come to shortly) for this problem, which we will henceforth refer to as the *15-15 variant*, in which Christians and Turks are sometimes replaced by people wearing black or white, by different coloured pawns, or by well-behaved children and rascals...



Figure 3: 15-15 variant: by decimating every ninth person, the first 15 eliminated will be grey. The first five are indicated by arrows.

Bachet concludes his 23rd problem by returning to the Josephus problem:

qui se font par les nombres. 177
Turcs s'en iroyent, & les 10. Chrestiens demeureroient.
Or comme j'ay touché en la préface de cette œuvre,
c'est par ceste inuention que Iosephe se sauua tres-subti-
lement dans Jotapata ainsi qu'on recueille euidentement
des paroles d'Egesippus touchant ce fait au 3. Liure de
la guerre de Hierusalem. Et bien qu'il ne particularise
pas assez ceste action, toutesfois par ce qu'il dit nous
nous pouuons imaginer comme le tout se passa. Car ain-
si qu'il raconte, il y eut 40. Soldats qui se sauuerent avec
Iosephe dans le lac si bien qu'à conter ledit Iosephe ils
estoient en tout 41. Partant supposons qu'il ordonna
que contant de trois en trois, on tueroit tousiours le troi-
siesme: il est certain que procedant de la sorte, tu trouueras
en fin par la regle donnée en la demonstration, qu'il faut
que Iosephe se mit le trente-vniesme apres celui, par le-
quel on commençoit à conter, au cas qu'il visast à de-
meurer en vie luy tout seul. Mais s'il voulut sauuer un
de ses compagnons, il le mit en sa seizesme place, & s'il
en voulut sauuer encor un autre, il le mit en la trente-
cinquesme place.

The text reads:

As I alluded in the preface to this work, it was this invention that so subtly spared Josephus in Jotapata, as we gather from Hegesippus's testimony in the tome 3 of the War of Jerusalem. And though he does not go into enough detail, from what he says we can nevertheless imagine what occurred. It is thus that he recounts how 40 soldiers escaped to the lake with Josephus, and that with the said Josephus there were 41 men in all. Thus, let us suppose that he ordered that every third man counted would be killed: it is certain that in doing so, you will find, in the end, by the rule given in the presentation, that Josephus had to stand in thirty-first place after the first man counted if he intended to be the only man to remain alive. But if he wished to save one of his companions, he had to put him in sixteenth place, & and if he wished to save another, he had to put him in thirty-fifth place.

This passage suggests that it was Bachet's choice to count in threes (he states in the previously cited preface that the n th value is not given by Hegesippus). While this value is not specified in the *War of Jerusalem*, according to Bachet this work contained Josephus's elimination protocol, albeit with few details, but sufficient to confirm that Josephus did not simply place himself at the mercy of divine Providence.

As a side note, and contrary to what is sometimes argued, it should be pointed out that as far as we can make out, Bachet did not mistake the number of men (there were indeed 41) nor the solution to the problem (16 and 31). These values are equally correct in the first edition of 1612.

Bachet's work was republished much later on, in 1874 (third edition), annotated and commented on by A. Labosne. A later edition (the fifth), published in 1884, is available on the website of the Conservatoire numérique des Arts et Métiers.¹²

In this fifth edition, a note from Labosne, added to the preface, warns readers against an erroneous interpretation of the work of Josephus, whose fate had been subject to chance and Providence. However, like Lucas, Bachet concentrates on Hegesippus's version:¹³ it seems only natural to seek this particular version of the text, which as far as we can tell has no reason to be less accurate than any other.

III – HEGESIPPUS

Gallica has published several versions of *The Hegesippus*, including a handsome manuscript edition¹⁴ from the 13th century. The passage is located on the reverse side of leaf 77:

12. Page on [CNUM](#).

13. This point is not developed here, but adds to the general confusion: the *War of Jerusalem* (*De excidio urbis Hierosolymitana*), written around 370, was not written by Hegesippus, who lived in the 2nd century, but probably by Saint-Ambroise de Milan (this thesis is not unanimously supported and some identify the *pseudo-Hegesippus* as a certain Isaac).

14. Page on [Gallica](#).

ut minueret numerū repugnantū. Cōmittam' inquit sortē mortē
 di ordinē. ut se nemo subtrahat. cū sortē cōueniat uniuersos. Sortes
 istiusmodi cōdictio sit ut his q̄ sortē exierit. ab eo q̄ seq̄t' interimatur.
 Itaq; fere. ut sortē unūquēq; nō p̄pria uoluntas mortē adiudicet. Sicut
 q̄ unūq; sub sortē iudice. ⁊ ex sortē sceleris ⁊ liber captiuitatis :
 ut nō alieno arbitrio celeret. nec declinet suo mortē futurā. Nemo
 reculare poterit euentū. quē uel casus intulerit. ut dī uoluntas
 signauerit. fidē fecit oblatio. unūsortē adsens' sortē addeunt. Quisq;
 sortē ÷ designat'. paratā seq̄nti necē p̄buit. Itaq; accidit. ut intemp
 tal reliq̄s. ioseph' cū altero superet necē. Oanielus necessario. ut aut

A French translation by I. Millet of Saint-Amour is available on Google

LIVRE III.
 selon les saintes loix, qui nous ont esté laissées par noz pe-
 res: Soyez certains que la liberté nous en est maintenant de
 beaucoup plus grande, veu que noz saintes ordonnances
 nous defendent ceste maniere de mort.

Comme les Iuifs, estans avec Iosephe, s'entretenoient
 & luy mesme, avec l'un des siens, vint es mains de
 Vespasian.

Chapitre 18.

Elles & semblables raisons alleguoit Iosephe à ceux, lesquels s'estoient retirez en la cisternne avec luy, pour leur oster de la fantasie ceste folle volonte de mourir. Mais iceux ayas vne fois pour toutes delibere de trouuer fin à leur vie, ne luy pouuans resister par paroles, se renegerent tous à l'entour de luy, chacun d'eux tenant l'espee toute nue, & le menaçant en cas qu'il ne s'y voulsist accorder, de le faire trespasser sur le cháp, & sans plus longuement attendre. Toutefois par cela il ne fut beaucoup estonné, ains les destourna de leur temerite entreprinse par l'autorité de capitaine qu'il estoit, accompagné de magnanimité de courage, laquelle luy estoit familiere à toutes heures: l'autre, par vn asseuré & seuer regard: de cestuy il retenoit la dextre: il refroidissoit l'ardente colere de l'autre, l'adoucisant par admonitions salutaires: tellement qu'en diuerses manieres mitiguoit la desreiglee fureur de tous ses copagnons: lesquels, n'obstant que par desfortune il fut tombé en fort bas estat, malgré bon gré le fort malheureux luy portoit encorres telle reuerence, que bien il meritoit. A cause de quoy petit à petit ils osteroient leurs mains de sus luy, remettant les epees en leurs fourreaux, combien qu'ils ne se

D'EGESIPPE. 157
 ne se pouuoient diuertir de leur mauuaise volonte. Quoy congnossant, & que luy seul estoit assiege de tant d'hommes, fortune l'auisa de diminuer le nombre, leur tenant tel propos. Compagnons, ie ferois d'opinion, si vous en estes d'accord, que le sort soit iuge de l'ordre que nous deuous tenir à trouuer la fin de noz vies, veu qu'il tombera sus chacun de nous egalemt, & qu'il n'y aura ame qui s'en puisse exempter. Et voila come nous ferons, celuy, à qui premierement le sort viendra, sera tué par son suyuant, & ainsi des autres consecutiuent, de sorte que le mesme sort seul condempnera vn chacun de nous à la mort, non pas nostre propre volonte. Parquoy il faut qu'un chacun de nous obeisse à l'arrest donné par la chance, comme à celuy de nostre iuge, nous garantissant d'offencer nous mesme de noz mains, ensemblement de tomber au pouuoir de l'ennemi, pour estre captifs, nul de la compagnie pouuant auancer le trespas de son compaignon, ny euitier le sien, estant par ce moyen retranchée la puissance d'euitier les euenemens, que le hazard ou Dieu nous apportera. Ceste offre leur fut tant agreable qu'ils ne se doutèrent aucunement de ruse, quelle qu'elle fut, aincoys y donnerent prompt consentement d'une voix: tellement que sans contradiction ils s'entretenoient les uns les autres, comme le sort leur tomboit sus, le suyuant mettant à mort celuy qui le precedoit. De sorte que tous occis Iosephe demeura seul avec son copaignon, estant necessaire qu'il mourut, comme les autres, ou que du moins il fut contaminé du sang de son prochain, qu'il espancheroit. A quoy voulant obuier, luy conseilla quitter & renoncer au sort, comme il fit, dont il euita le combat, & sortirent ensemblement du creux de la cisternne, pour estre acconduits à Vespasian par Nicanor. Adonq vous eussiez peu voir de

R

LIVRE III.
 tous les quartiers du camp les Romains accourir, desirans voir le pauvre Iosephe mis en si piteux estat & termes, estat nagueres morté en si haut honneur, capitaine general d'une si belle armée, & pais de telle estendue: Les autres y venoient pour se mocquer de luy & de sa prison: & les plus sages s'esmeruilloient de l'incostance & soudaine mutation des affaires humaines. Aucuns, estans encorres mieux aises, lamentoient en leur cœur la calamité, en laquelle ils voyoient tombé ce grand personnage, considerans que come luy, ils pourroient vn iour estre miserables & desnuiez de saueur: entre lesquels estoit le ieune prince Titus, qui selon son acoustumee douceur & mansuetude estoit fort marié, voyant celuy, qui autrefois & non long temps par auant auoit esté si braue capitaine, estre tombé en la mercy de ses auersaires, attendait la sentence qu'ils bailleroient contre luy, n'ayant aucun espoir ny assurance de son salut. Si grand pouuoir ont petites occasions aux affaires beliques, que pour bien peu de cas, & en peu d'heure, les plus grans seigneurs sont rengez avec les plus petis soldats, les astigez & de cōdition basse eleuez à grandes richesses & honneurs. A raisō de quoy, les principaux estoient ceux qui conseilloyent qu'il fut traité le plus doucement, mesmes entre les autres Titus fut intercesseur de luy sauuer la vie, Vespasian le faisant mettre en prison avec seure garde, de peur que gagnant au pied il trouuast moyen de se sauuer.

L'amiée de Vespasian à Ptolemais & Cesarée: puis comme il enuoya partie de ses gens passer l'hyuer en lieu commode.

Chapitre 19.

Quel-

Books:15

The text reads:

Companions, if you are in agreement, I am of the opinion that fate should be the judge of the order we must uphold as we meet our deaths [...] the man first chosen by fate will be killed by the next man chosen, and so on in succession [...] This suggestion was all the more appealing to the men for none suspected any ruse whatsoever, and with a single voice they

15. [Page](#) on Google Books.

promptly assented: without opposition the men killed one another as fate befell them, the following man executing the former. Thus it came to pass that when all were slain Josephus alone remained with his companion, and so it was that he should die, like the others, or at least taint his hands with the pouring blood of his kin.

Once again, we lose the scent. While the text is ambiguous (the word *fate* and *ruse* are both used), there is no mention of the soldiers being arranged in any particular way, nor of Josephus choosing his *fine position*. However, the word *ruse* makes an appearance, which is not the case in the *non-Hegesippus* versions of *The Jewish War*. The two important passages, referring to the system of drawing lots and the fact that the soldiers do not suspect any ruse, are also present in a Latin manuscript version mentioned above (although the hypothetical ruse is not mentioned in such explicit terms):

Sortis istiusmodi condicio sit ut his qui sorte exierit ab eo qui sequitur interimatur.

That the condition of this drawing of lots is that he on whom the lot falls will be killed by the next such man.

Fidem fecit oblatio et universorum adsensus sorti adquevit.

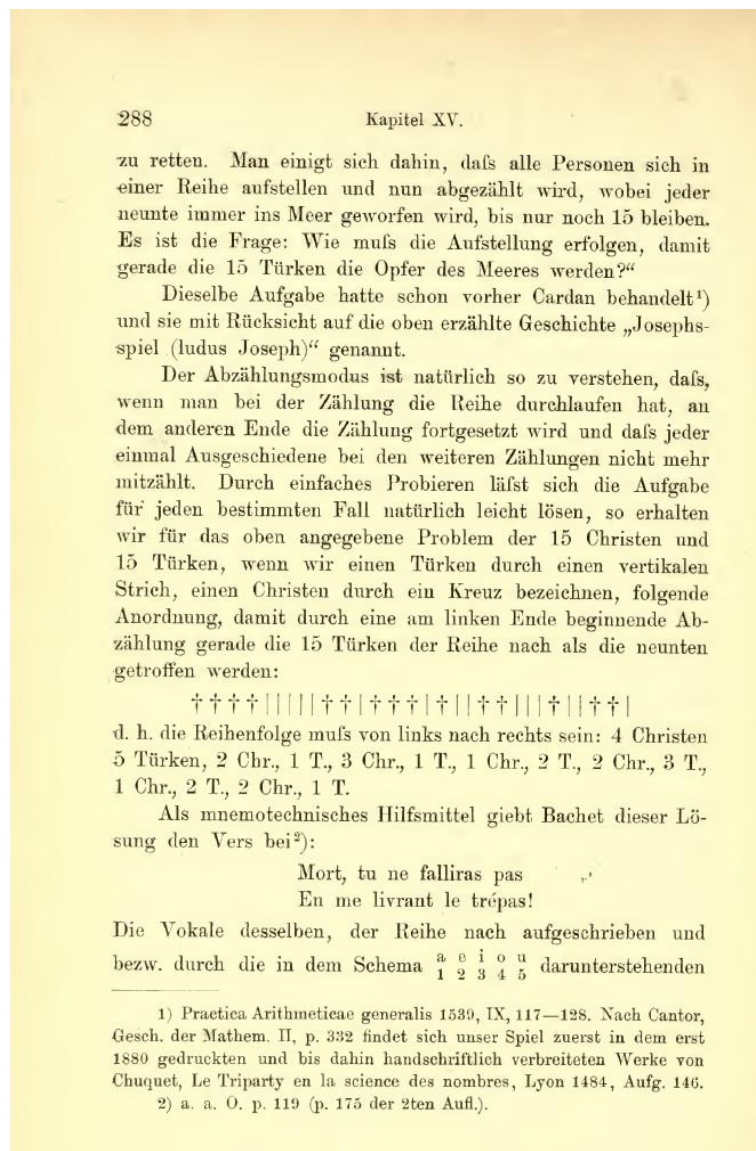
The suggestion inspired confidence and all agreed to accept the drawing of lots.

Could it be that Bachet came up with the problem, inspired by the simple evocation of a possible ruse in the Hegesippus's translation? The first edition of *Problème plaisants et délectables* was published in 1612. There are stylistic differences in the passage in the preface, but the content is identical. And the problem (the 20th in the edition of 1612) is identical. For the time being, then, the earliest trace of Flavius Josephus the number cruncher is found in Bachet in 1612.

IV – TREATY OF MATHEMATICAL GAMES

Wilhelm Ahren's treaty on the history of mathematical games, published in German at the turn of the last century, and as far as we know never translated, fleshes out the story. Chapter 15 explores *The Josephus Game*. The first edition

of *Mathematische Unterhaltungen und Spiele* (Mathematical Games and Puzzles), published in 1901, is available online:¹⁶



Ahrens notes in the second paragraph:

The same problem had already been dealt with by Cardano, who had christened it "The Josephus Game" (Ludus Joseph), in reference to the tale recounted above.

The footnote provides clarification:

According to Cantor, our game is mentioned in the works of Nicolas Chuquet, which were first printed at the beginning of the 1880s and until then circulated in manuscript form.

16. [Page](#) on OpenLibrary.

V – CHUQUET, CARDANO AND TARTAGLIA

Among the sources cited by Ahrens, the oldest is Nicolas Chuquet. Chuquet wrote his manuscript in 1484, and a copy is conserved at the French Bibliothèque Nationale. The story of this text is an interesting one: it was used by Estienne de La Roche in the 16th century to compose his own treaty, *Larismetique*, published in 1520¹⁷ (some say that Estienne de La Roche shamefully copied passages from Chuquet's manuscript without citing his source). Chuquet's work was then "forgotten" for over 300 years until being rediscovered by Aristide Marre, who reprinted it in 1880 and 1881 in the *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*.¹⁸

The passage we are interested in is found in the *Problèmes numériques faisant suite et servant d'application au Triparty en la Science des nombres de Nicolas Chuquet* and forms Problem 146 (page 453). This is the very same problem recounted by Bachet, although the protagonists' religions are not the same. The problem involves counting in nines and selecting 15 preselected people from a group of 30. Figure 4 is a reproduction of the illustration that accompanies Problem 146 in Chuquet's manuscript (the illustration was not reproduced in the 1881 publication).

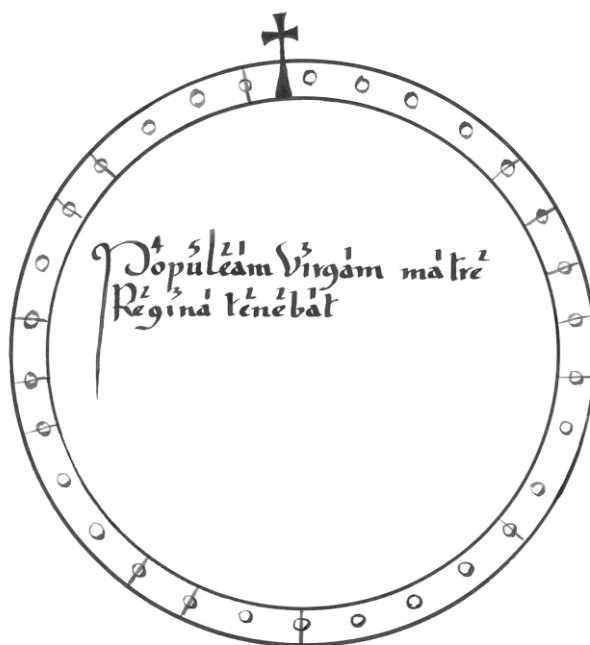


Figure 4: Reproduction of the drawing illustrating Chuquet's Problem 146. The circles indicate the position of those who will be saved, and the crossed-out circles that of those who will perish. The illustration reproduced the Latin mnemonic used by Chuquet to

17. *Larismetique* can be consulted on [Google Books](#).

18. The 1880 and 1881 publications are available online ([here](#) and [here](#)).

remember how the 30 individuals should be arranged (Lucas used an almost identical phrase at the end of the 19th century): Populeam virgam matre regina tenebat.¹⁹

Chuquet, however, does not mention Josephus or any similar problem. It is the 15-15 problem that interests him.

Another source cited by Ahrens is a work by Cardano, *Practica Arithmeticae*, published in 1536. Between the first edition of the *Mathematische Unterhaltungen und Spiele* in 1901, and the last, in 1918, Ahrens refines his position by stating that:

The game of Josephus – "Ludus Joseph" as it is called – was first mentioned in Cardano's Practica Arithmeticae.

Cardano's text can be found in the edition of 1536²⁰ or in a later edition (the *Opera Omnia Opera Omnia*²¹). The same text appears in both, and the extract that interests us is found in the middle of paragraph 18 at the end of Chapter LXI, *De extraordinariis & Ludis*.

19. "The queen mother held a poplar branch."

20. [Page](#) on Google Books.

21. [Page](#) Università degli Studi di Milano.

iori, & minori æqualiter, auget magis supra maiorem quam supra minorem, & ita remotis fraudibus, & scientia æquali existente, impossibile quasi esset pauperem non vincere, verum pauperes aliquando impedit timor, aut letitia, diuides autem non cum tanto affectu ludunt, & ideo securius, &c.

8 Et ex his dicamus de ludis aliquid & sunt memoratiui, veluti quando quis vult intelligere numerum excogitatum, facit vt adiciat medietatem quod si non potest facit vt compleas, deinde facit iterum adicere medietatem, & si non potest facit vt compleas, deinde proicit. 9. & quotiens proicit totiens 4. inuenitur in numero excogitato, & causa est quia proportio 9. ad 4. est composita ex duabus sexquialteris, potest igitur accidere quadrupliciter & dabo tibi exemplum in 4. modis, cogitet primo 17. adde dimidium fit $25\frac{1}{2}$, & quia est fractio comple fit 26. adde fit 39, proice 9. quater igitur duc 4. in 4. fit 16. & quia fractio fuit primo loco igitur habuit 17. addendo vnitatem ad 16. cogitet secundo etiam 18. adde dimidium fit 27. dimidium fit 40. $\frac{1}{2}$ comple fractionem fit 41. proice 9. quater duc 4. in 4. fit 16. & quia fractio fuit secundo loco adde 2. fit igitur 18. & ita cogitauit cogitet tertio etiam 19. adde dimidium fit $28\frac{1}{2}$ & quia habet fractionem comple fit 29. adde dimidium fit $43\frac{1}{2}$ comple fit 44. proice 9. fit hoc quater duc 4. in 4. fit 16. & quia fractio fuit primo & secundo loco adde 3. fiet numerus cogitatus 19. cogitet quarto 20. adde dimidium fit 30, adde dimidium fit 45. diuide per 9. exit 5. duc in 4. fit 20. & quia nulla fuit fractio, idem dices quod cogitauit 20. & hoc est generale. Et similiter cognoscunt annulum vbi fuerit abscondus inter plurimos homines. ei in qua manu, digito, & articulo. Et similiter inter tres res quis habeat distinguunt, cum 18. tabulis, aut lapillis. Et similiter cognoscunt cartam numero cogitam per tertiam diuisionem in quatuor, & similiter ponunt quotquot voluerint tabulas in circuitu & excipiunt albas dimittentes nigras, fit autem in 15. tabulis albis & totidem nigris sed potest fieri in quolibet numero, & dicitur ludus Ioseph, qui cum hoc sociis vt dicunt per sortem vt illi putabant mortem imminere, ipse quia inopia premebatur cum socio tantum seruatus est, & disponunt quotquot lapillos in circuitu & per duas contrarias numerationes faciunt exire cogitatum ex illis, & hic inter ceteros non intelligentibus est mirabilis, licet sit res simplex, & sunt ludi mentales & sunt vt vnus habet 1. 3. 6. potestate: alius 2. 4. 5. & vadant ad 100. aut vnus 1. 3. 5. 8. 9. alius 2. 4. 6. 7. 10. & vadant ad 100. qui perfecit vincit, & sunt magnæ inuentionis, & ego inueni æquitando & sine aliquo auxilio cum socio potes ludere & memoriam exercere, & ad sunt loca fallaciarum, & triumphi, & vacua in vnoquoque, vt non minor sit ludus sciacorum mentali quantum igitur longam & inutile esset infinitas numerorum differentias in ludis referre ob hæc pertransimus

Tom. IV.

ita tamen vt scias horum duorum vltimorum cum qui fit per 1. 3. 6. minorem alium qui fit per. 1. 3. 5. 8. 9. maiorem appellari, fit etiã dando. 6. 2. 1. vtrique lusori imaginando fritillum & huius memorie & ingenij non est finis, ita vt etiam cum ipso fritillo, non parum sit optime luisse, fit etiam ludus transitus, fit & intercipiens, fit & ludus proportionalitatum dispositus in fine Arithmetice Fratris Iordani, sed de his factis.

Pertinet & ad extraordinarias quæstiones adicere quasdam interrogationes vt panis in valore frumenti solidorum 100. fit vnciarum 9. quando valet solidos 140. quot debet fieri vnciarum, duc 100. in 9. fit 900. diuide per 140. fit vnciarum $6\frac{3}{7}$: & ideo est ac si diceret si 140. fit 100. quid fiet 9. & patet quod est conuersa in operatione ad alias est tamen in similibus regula generalis.

C A P V T LXII.

De Datis.

DAta dicuntur cognita cum ignota ex notis cognoscuntur, veluti cognosco quidem 10. & quid sit medietas, igitur cognosco etiam quid sit medietas. 10. quæ est 5.

Cognoscere qua dupliciter dicitur & omnibus his modis dicitur datum primo modo perfecte & nominatim, & hoc modo cognoscimus 7. & omnem numerum integrum, vel fractum, aut perfecte non tamen nominaliter veluti cum cognosco 32. 7. aut omnem quantitatem irrationalem, tertio modo cum cognoscimus secundum propinquum veluti cum scio cordas arcuum vel motus cœlestes, nam non sciuntur præcise nisi admodum pauca & hoc modo dicimus quod hoc cognitum insensibiliter differt ab incognito quod est vera quantitas, & hoc vtuntur Astronomi, & præcipue Ptolomæus, quarto dicimus datū inter duas quantitates notas veluti cū dicimus quod proportio circumferentiæ ad diametrum est minor quàm 12. ad 7. & maior tripla & $\frac{10}{7}$, & hoc est dicere quod est maior quam 223. ad 71. & hic modus est quo vtitur Ptolomæus ad constituendam cordam vnus gradus, per cordam gradus & dimidij, & per cordam arcus trium partium ex quatuor vnus gradus, & hoc vtitur Ioannes Monte Regius contra Nicolaum de Cusa, & de his datis præfertur quidam liber Euclidi ascriptus.

His visis in quolibet quatuor modorum, cognoscimus aut differentiam, aut quantitatem, aut proportionem, & tunc vel differentia cognoscitur ex duabus quantitatibus cognitis, aut ex duabus proportionibus cognitis, aut ex quantitate & proportionem cognitis, & similiter aut quantitas cognoscitur ex quantitate & differentia, aut quantitate & proportionem, aut ex proportionem & differentia, & similiter aut cognoscimus proportionem est duabus quantitatibus, aut ex quantitate & differentia, & ita sunt 8. modi, & fit cognito, etiam in trigonis &

K 3 quantitatibus

What follows is a short passage in French about the decimation games:

[...] in the same way, we place as many tablets as we like in a circle, selecting the white ones and putting the black ones to one side; let us say that there are 15 white boards and as many black ones, but the statement holds for any number; we know Josephus's game, in which Josephus inflicted death on his companions in such a way that the latter believed they were at the mercy of fate, while he, catching his peers unawares, was spared along with one of his companions; we arrange as many little stones as we like in a circle and once two have been counted we remove the third; this game seems admirable to those without judgment, although it is simple...

Here Cardano succinctly sums up the statements of the two problems: the 15-15 version with black and white boards, and the Josephus game. The rules he gives are surprisingly incomplete: while he mentions the fact that it involves counting in threes²², he does not specify the number of people (which is nonetheless the only piece of information stated in *The Jewish War*), and therefore does not give the solution to the problem either.

We have already seen that the 15-15 version of the decimation problem is older than Cardano's work, since Chuquet mentions it. While it is impossible to prove, it seems possible that the version of the problem associated with Josephus was probably also in circulation before Cardano wrote it down. If the reverse were true, he would have probably quoted at great length from Josephus's work and would no doubt have gone into more detail (number of people, solution, etc.).

@@@@@@

Before concluding this survey of the Josephus problem throughout the ages, mention should be made of Niccolò Tartaglia's work *General trattato di numeri, et misure*, published between 1556 and 1560, which includes the problem in its 15-15 variant. It is more or less certain that Tartaglia was aware of Cardano's work when he was writing, because he had anxiously consulted the latter to see if the publication contained the method for resolving third-degree polynomial equations, which he had communicated to Cardano but did not wish to see published.²³

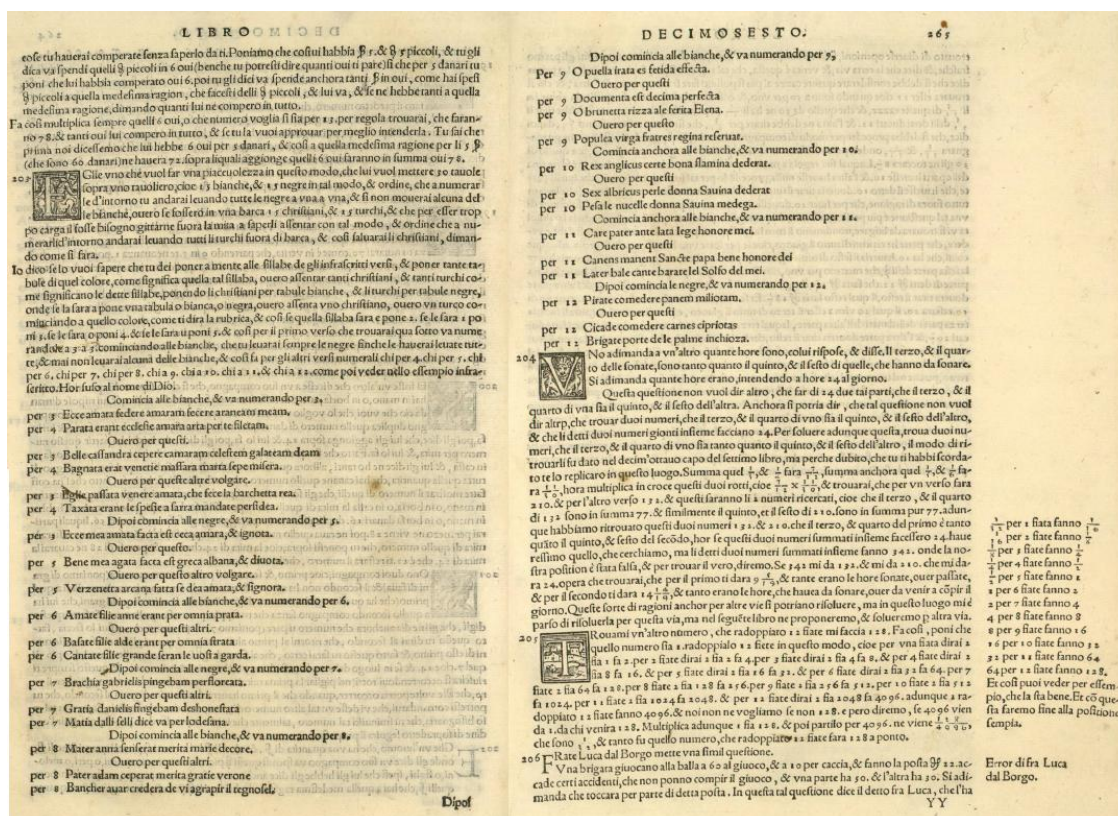
22. Cardano's work thus contains the earliest mention of counting in threes that we have found.

23. It would have been published, however, in Cardano's *Ars Magna* (1545), after Cardano claimed that Tartaglia was not the *first* inventor of the resolution method for these equations but had been preceded, again according to Cardano, by Scipione Dal Ferro.

Tartaglia's text has been digitised and put online by the University of Strasbourg:²⁴ the decimation problem is referred to in paragraph 203. Here is a translation of the opening of the paragraph:

If someone would indulge me by arranging 30 pawns – 15 white ones and 15 black ones – on a draughts board, in such a way that by counting from the first to the last, you will remove the blacks one by one without touching any of the whites. It is as if there were an overloaded boat containing 15 Christians and 15 Turks, and necessary to throw half the passengers overboard, and one had to know how to seat and order them so that by counting one could throw all the Turks overboard and save all the Christians: the question is how to do so.

I say that if you wish to know how, you must bear in mind all the syllables of the verse below and arrange as many black or white pawns (or seat as many Christians) as indicated by the aforementioned syllables.



Tartaglia then states the vowel/number equivalence we have already mentioned, and ends with a comprehensive list of mnemonics to recall how to arrange the 30 people depending on whether one counts in threes, fours, etc.

Although he does not deal with the Josephus game, Tartaglia seems to have treated the problem in more depth than Cardano, or at least more

24. [Page](#) University of Strasbourg.

systematically, and makes his contribution by suggesting new mnemonics for the variants of the game.

VI – CONCLUSION

Any conclusion is unsatisfying given the number of variants of the historical Josephus text and books of arithmetical puzzles that have covered the problem. Each discovery almost systemically leads to new avenues to explore.

However, without saying anything original, we can at the very least confirm, drawing on the texts, that neither the “classical” versions translated from the Greek nor the pseudo-Hegesippus version of *The Jewish War* contain proof that Josephus was a lover of arithmetical problems.

The case of the Slavic version is less straightforward. It seems likely, however, that the Slavic then French translators were influenced by their existing knowledge of the decimation problem and integrated it into their story.

As for the paternity of the Josephus problem, notwithstanding the fact that it is mentioned by Cardano in his *Practica Arithmeticae*, the bulk of the work, as we have seen, is attributable to Bachet and his *Problèmes plaisants et délectables* published in 1612.

Cardano’s testimony leaves the impression that, unlike other problems for which the author can claim paternity, the Josephus problem emerged from a complex web of decimation problems (such as the 15-15 version with Turks and Christians) well before it was published. Cardano was thus probably reporting a problem that was already well known. If the opposite were true, he would almost certainly have given more details. The paternity of the Josephus problem, from this point of view, probably does not lie with either Bachet or Cardano.

As for decimation problems in general, and in particular the 15-15 problem (division into two groups) that has cropped up several times, their origins are much older than the one we have indicated here (Chuquet, 1484), for this type of problem appeared in texts as early as the 12th century.



(September 2012)

(translated in English by Helen Tomlinson, published June 2014)

Annexe: IT solution

It would be unthinkable to conclude this article without proposing a modern solution to the Josephus problem. The following Python function indicates the safe position in a scenario with n people and counting in ps . The third, optional parameter indicates the number of people to be saved. The answer gives the list of positions of the people in the circle.

```
def josephus (n,p,m=1) :  
    c=list (range(n))  
    pos=0  
    while len(c)>m :  
        pos= (pos+p-1)%len(c)  
        del c[pos]  
    if len(c)==1 : return c[0]+1  
    return tuple(v+1 for v in c)
```

We can thus check that place 31 is the one Josephus should occupy:

```
>>> josephus_(41,3)  
31
```

It is immediately clear that if you find yourself in a Roman legion of 6,000 men facing the same fate as Josephus's companions:

```
>>> josephus(6000,3,2)  
(3949, 5864)
```

you and your best friend must choose the places 3949 and 5864.

@@@@@@

The solution to the problem of the Turks and Christians is obtained by asking for the list of the 15 survivors out of a total of 30 people and a decimation counted in nines:

```
>>> josephus(30,9,15)  
(1, 2, 3, 4, 10, 11, 13, 14, 15, 17, 20, 21, 25, 28, 29)
```

Adding the following function will reproduce the arrangement specified by Bachet:

```
def placement(seq,n) :  
    s=[[1 in seq,1]]  
    for i in range(2,n+1) :
```

```

    if s[-1][0]==(i in seq) : s[-1][1]+=1
    else : s.append([i in seq,1])
    return tuple([(T',C'][i],j) for i,j in s)

```

We therefore obtain:

```

>>> arrangement(josephus(30,9,15),30)
(('C', 4), ('T', 5), ('C', 2), ('T', 1), ('C', 3), ('T', 1), ('C', 1),
('T', 2), ('C', 2), ('T', 3), ('C', 1), ('T', 2), ('C', 2), ('T', 1))

```

The order is familiar: 4 Christians, 5 Turks, 2 Christians, and so on.

@@@@@@@

Lastly, it is possible to resolve the cat problem posed by Dudeney:

```

>>> n=13
>>> josephus(n,n)
8
>>> n+1-josephus(n,n)
6

```

The eighth mouse will be the last to be eaten. In order to save the white mouse for last, the cat must start counting from the mouse located six places after the white mouse.



I wish to thank the éditions H&K for their permission to use figures from *Divertissements mathématiques et informatiques* (2011).

My thanks also to Céline Michaud for her advice on the English translation (Dudeney), to Alexandre Moatti for his translation from the Italian (Tartaglia), to Nima Yeganefer for his translation from the German (Ahrens), to Sergueï Martemianov for his advice on Slavic, and to Anne-Sophie Traineau-Durozoy for her translation from the Latin (pseudo-Hegesippus, Cardano, ...) and her precious help in general.