Reflections on the Motive Power of Fire by Sadi Carnot

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At the start of the nineteenth century, steam engines were omnipresent in the western industrial world, especially in England. Many engineers tried, in a practical way, to improve their often very limited efficiency: it was not until 1824 that Sadi Carnot developed a true scientific theory of heat and its application to heat engines. Nicolas Léonard Sadi Carnot (1796-1832), former *École Polytechnique* student, is as such one of the pioneers of thermodynamics: at 28 years old he designed the ideal theoretical thermodynamic cycle that is still used today as a point of reference for heat engines, the "Carnot cycle". Sadi Carnot was the son of physicist Lazare Carnot, known as "the great Carnot"; not to be confused with his nephew, Marie François Sadi Carnot (1837-1894), who also had the same common first name Sadi, and was president of the French Republic from 1887 until his assassination in 1894.

Figure 1: Sadi Carnot (1796-1832). *His portrait in Polytechnique uniform, painted by Boilly in 1813.*

Heat Engines : An Economic Issue

Sadi Carnot wanted to crack the secrets of heat and its use in machines that supplied motive power. He began his reflections (which were rather philosophical to begin with) by his analysis of transforming heat into motion in nature. He was in fact the first to consider the equivalence between heat and movement and he was impressed by this available natural power that was just waiting to be exploited:

> *It is in this huge fuel tank that we can draw the necessary motive force to our needs; nature, while offering us fuel from all sides, has given us the ability to create at all times and in all places heat and the ensuing motive power. Developing this power, appropriating it for our use, is thus the purpose of heat engines.*

The economic issues related to the increase of efficiency of heat engines are one of the main motivations for Sadi Carnot. The steam engine allowed the development of the great powers of the nineteenth century, particularly in England. Sadi Carnot writes in page 4:

> *Taking away today from England her steam engines would be to remove at the same time coal and iron; this would dry up all sources of wealth, destroy all means of prosperity and would annihilate this colossal power.*

The Limitations of the Heat Engines

One of Sadi Carnot's main concerns was whether there was an achievable maximum in terms of efficiency to transform heat into motive power, to which he refers in page 6:

> *It has often been discussed whether the motive power of heat is limited or not.*

His approach is original because Carnot not only sought to improve steam engines but he also tried to find a general scientific theory for heat engines, just like a developed mechanical theory:

> *We must establish applicable reasoning, not only to steam engines but to all heat engines imaginable.*

Heat, Calorific (Heat value), Motive Power and Work

Heat is energy exchanged between two bodies spontaneously at various temperatures, so as to reach a thermal equilibrium. Measured in joules, it is the result of the propagation of thermal agitation of particles step by step at a microscopic level. At the time of Sadi Carnot, the terms "*heat*" and "*calorific*" were synonymous and referred to this energy that was regarded then as a kind of fluid flowing from hot bodies to cold bodies. Carnot also used this principle of heat (calorific value) in his *Reflections*. Motive power – the term used by Sadi Carnot in 1824 - is not a power in today's sense but instead an energy that allows movement, also known as "*work*" and noted as "*W*"[1]. Sadi Carnot in general expressed this size in cubic metres of water raised 1 metre high (or per unit of 1000 kilogram-metres, the work unit being often used in the nineteenth century.) and corresponding to a work of around 10,000 joules ($W = M \times g \times h$ = 1000 kg * 10 m/s² * 1 m = 10 000 J).

It was thought for a long time that steam engines "consumed" heat (calories) such as fuel; Sadi Carnot however puts an end to this hypothesis on pages 9 and 10, explaining scientifically from the simple analysis of a steam engine that heat is not consumed but is instead simply transported from a hot source (the firebox where the combustion occurs) to a cold source (the cold water of the condenser) and it is this transportation which generates the motive power:

> *In steam engines the production of motive power is thus owed not to an actual consumption of heat but to its transportation from a hot body to a cold body, that is to say, to its restoration of balance, the assumed balance disturbed by all causes whatsoever, by chemical action, such as combustion or other.*

The Steam Engine

The steam engine appeared in the middle of the eighteenth century, the first patent was filed by James Watt (1736-1819) in 1769. This was a heat engine that allowed the transformation of the heat produced by burning

1. It was physicist Coriolis (1792-1843) who proposed in 1829, in his *Calcul de l'effet des machines*, the term "work", formally known under various terms (amount of action, power, mechanical work,...).

coal into mechanical motion using steam - in general to turn a wheel. Carbon is burned in a firebox connected to a boiler filled with water. The water evaporates and gives rise to steam that is sent to a cylinder pushing a piston that leads to a connecting rod allowing the rotation of a flywheel. The steam is then condensed in a condenser in which cold water circulates, generally derived from a nearby waterway.

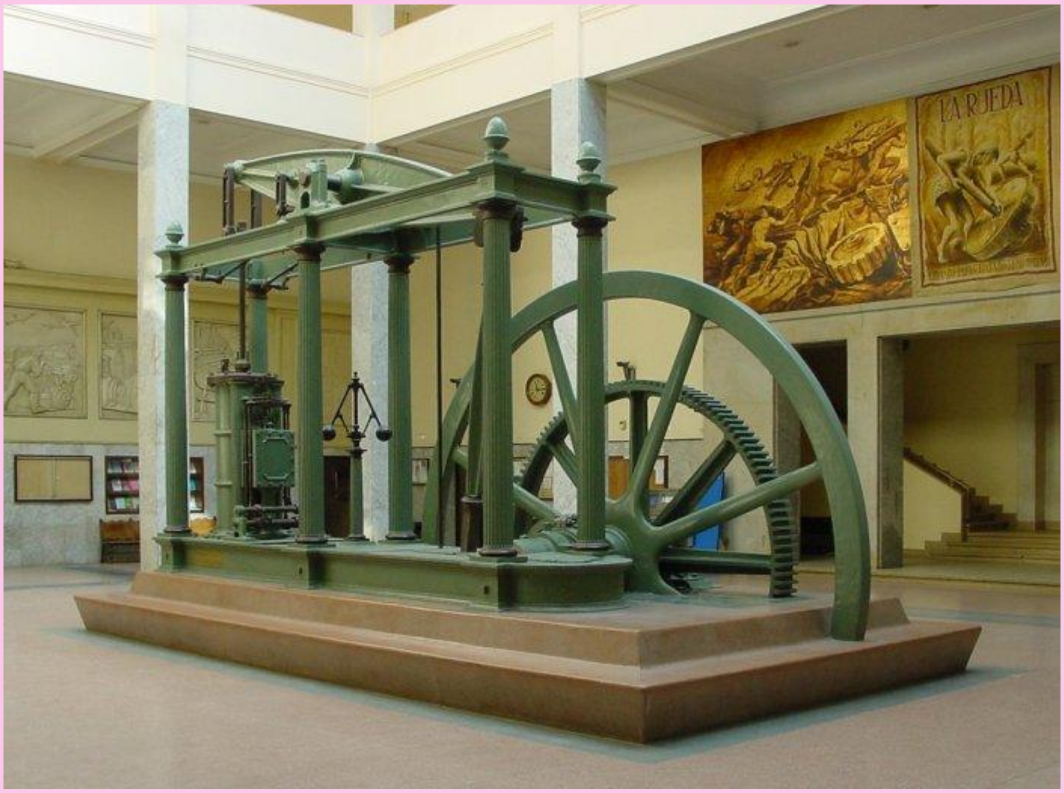

Figure 2: ***The Watt steam engine (1769).*** *Notice the centrifugal regulator (sliding balls on a vertical axis) which ensures the smooth functioning of the engine, independent of variations of power solicited at the outlet. (Image: Wikipedia, the lobby of the Superior Technical School of Industrial Engineers of the UPM (Madrid)*

Steam was certainly the most used fluid in heat engines at the time, but Sadi Carnot insists throughout his *Reflections* on the fact that any element can be used. For example, on page 10 he says:

> *Water vapour (or steam) is a way to achieve this power, but it is not the only way: all bodies of nature can be used for this purpose; all are subject to change in volume, contractions, successive dilations by alternatives of heat and cold* [...] *A solid body, a metal bar, for example, alternately heated and cooled, increases and decreases in length and may move the bodies fixed to its ends.*

Once the independence of the vector is demonstrated, Sadi Carnot clearly shows the reversibility of the transformations put in play as well as the equivalence between temperature difference and motive power:

Wherever there is a temperature difference, they may be the production of motive power. Conversely, wherever this power may be consumed, it is possible to create a temperature difference.

He then describes in pages 17 and 18 the different thermodynamic transformations that occur in a steam engine. Written in a more modern way, here are the three transformations:

1 – "Take heat from body A in order to form steam": **isothermal vaporisation** of water into steam at the temperature T_A thanks to a hot source (boiler fuelled by coal).

2 – "The steam having been received in an extensible capacity, such as a cylinder equipped with a piston, increase the volume of this capacity and consequently also that of the steam. Thus rarefied, it will decrease spontaneously in temperature": **adiabatic compression** (without external heat transfer) of steam in a piston: the lowering of the temperature T_A to T_B.

3 – "Condense the steam by connecting it with body B (...) until it is entirely liquefied": isothermal liquefaction of the steam in the condenser at the temperature T_B with the help of a cold source (in general nearby waterways).

Once again, Sadi Carnot tries here to give a scientific description of the different phenomena while remaining understandable - using simple words and without formulas. At the same time, he refers to the practice of machinery - from which he draws his theory: body A corresponds to the boiler and body B (cold body) corresponds to "the water for injection", taken from a nearby waterway and allowing condensation.

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It is on page 22 that he makes his main proposition, which will give birth to the famous Carnot cycle:

The maximum motive power resulting from the use of steam is also the maximum achievable motive power by any means whatsoever.

Here again we see Sadi Carnot's desire to develop the most general heat engine theory possible. To achieve this, Carnot assumes that perpetual motion is impossible, which had already been approved since the late eighteenth century by the Académie des Sciences in Paris under the influence of Laplace. However, he insists on the impossibility of perpetual motion by any means whatsoever: a system using heat or electricity, in the manner of a mechanical system, cannot result in a perpetual motion and thus create motive power *ex nihilo*. On page 23, he makes the first condition to be met in order to obtain maximum efficiency in a heat engine:

> *The necessary condition of the maximum is thus* that it takes place in bodies used to produce motive power of heat free of temperature change, caused by a change of volume. *Conversely, whenever this condition is fulfilled, the maximum will be reached.*

In other words, two bodies having different temperatures should never be in contact with each other as the heat exchanged during this contact would mean a loss of motive power. It is therefore necessary to make changes to temperature purely by compression and expansion, which likewise involves making isothermal compressions and expansions, which are barely possible in practice. For demonstrations of these propositions in pages 25, 26 and 27, Carnot makes use of calculus methods allowing him to carry out demonstrations by iterations and expand on the propositions to more general cases with several bodies.

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Sadi Carnot's father, Lazare Carnot (1753-1823), the great mathematician and physicist, was one of the founders of the École Polytechnique and worked a great deal on hydraulic machines. Also, his son does not hesitate to compare the hydraulic machines studied by his father to heat engines in terms of efficiency and maximum power:

> […] *the motive power of heat can be compared with reasonable accuracy to that of a waterfall: both have a maximum that cannot be exceeded* […]

Sadi Carnot attempts to connect the waterfall height of hydraulic machines with the difference of temperature in heat engines. He wonders about the possible relationship between the drop in heat and motive power at the bottom of page 28:

In the drop of heat the motive power undoubtedly increases with the temperature difference between the hot body and cold body; but we do not know if it is proportional to this difference. We do not know, for example, if the drop in heat from 100° to 50° provides more of less power than the similar drop in heat 50° to 0°.[2]

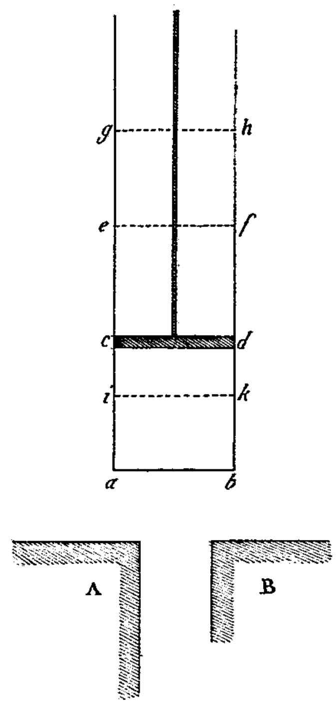
At that time there was still no mathematical relationship between the maximum theoretical mechanical power and temperature difference. Sadi Carnot then tries to make analogies with hydraulic machines in which motive power is proportional to the height of the waterfall, explaining the expression "drop in heat"[3].

THE CARNOT CYCLE

Sadi Carnot then presents a second (and much more rigorous) demonstration of his main proposition which states that motive power produced by a heat engine cannot exceed a certain limit. This limit is purely determined by the difference of temperature between the hot source and the cold source and is independent of the vector used (steam, water, alcohol, etc.). In other words, a heat engine can only provide work if it takes heat from a hot source (e.g., a boiler) only to then reject it towards a cold source (e.g., a waterway). This implies that monothermal engines cannot provide mechanical power; this proposition is the first statement of the second law of thermodynamics (see the boxed text at the end). This demonstration, made on page 18, is based on an ideal thermodynamic cycle produced in an ideal engine consisting of a piston in a cylinder: the Carnot heat engine is shown below.

2. The "Carnot efficiency", concept that was introduced after him, can answer this question: the efficiency from 100 to 50°C is 1 – 323/373 = 0,134, the efficiency from 50 to 0°C is 1 – 273/323 = 0,155.

3. Carnot seems to think that the "drop in heat" between a hot body and a cold body, i.e., the transfer of heat from a hot body to a cold body, creates a work. As the heat was then regarded as a kind of fluid, the analogy with the waterfall was almost perfect but actually wrong: the heat from the hot source is not equal to the heat rejected to the cold source because during the heat transfers (or "drop in heat"), entropy is generated and the work is not created by this heat transfer but by the transformation of the same heat.



<u>Figure 3</u>*: *Carnot heat engine *(figure I of the publication, page of figures at the end). A cylindrical vessel is provided with a movable piston, figured there at position 'cd', and which can move up and down so as to compress or expand the elastic fluid contained in the cylinder. The two bodies 'A' and 'B' symbolically represented at the bottom of the image are two isothermal bodies where A represents the hot source and B the cold source.*

The different thermodynamic transformations conducted in this ideal machine represent the Carnot cycle. The cycle is composed of four reversible transformations, which are described on page 33 and can be stated as follows:

1 – Isothermal expansion at the temperature T_A. On this occasion, an amount of heat $Q_A > 0$ is brought to the system (step 2 on page 33).

2 – Reversible adiabatic expansion (also isentropic, i.e., at constant entropy) where the temperature of the fluid decreases from T_A to T_B without exchanging heat with the outside (step 3 on page 33).

3 – Isothermal compression at the temperature T_B. This compression leads to the rejection of an amount of heat $Q_B < 0$ outwards (step 4 on page 33).

4 – Reversible adiabatic compression (also isentropic) where the temperature increases from T_B to T_A without exchanging heat with the outside (step 5 on page 33).

Concerning cycles 2 and 4, Carnot does indeed rely on the "experimental facts that best prove the change in temperature by compression or expansion" (such as for example the decline of the thermometer under a pneumatic engine where the vacuum is created), while stressing nevertheless that:

> *The temperature change caused in the gases by the change in volume may be regarded as one of the most important facts of physics because of many consequences that it entails and, at the same time, as one of the most difficult ways to clarify and to measure through decisive experiments.*

Nowadays, this experimental fact is well known: when air is compressed in a bicycle pump, it heats up; when an aerosol container is used, the fluid expands and cools the container.

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Ultimately, this cycle generates motive power (in other words, a work), equal to the amount of heat exchanged:

$$W= Q_A + Q_B$$

This ideal cycle assumes no particular fluid and provides the theoretical maximum of motive power because all changes in temperature are purely due to changes in volume (compression or expansion): at no point is there an exchange of heat through contact or a loss through friction. Furthermore, this cycle is reversible: by carrying out the transformations in the other direction and by providing the mechanical work, this is the ideal, which C refrigeration (or heat) cycle, which Carnot makes explicit on page 36:

> *[…] the result of the opposite operations is the consumption of the motive power produced and the return of heat from body B to body A […]*

The Carnot cycle went relatively unnoticed during his oral presentation to the Académie des Sciences in 1824[4], as well as in the work *Reflections*. It was not until ten years later, in 1834, that Emile Clapeyron (1799-1864) showed particular interest in Sadi Carnot's *Reflections* and understood its importance. Clapeyron then illustrated the Carnot cycle in a diagram that is now called the Clapeyron diagram in which the pressure is depicted according to the volume during the transformations.

4. Meeting of June 14th 1824, presentation by French Academy member Pierre-Simon Girard (1765-1835).

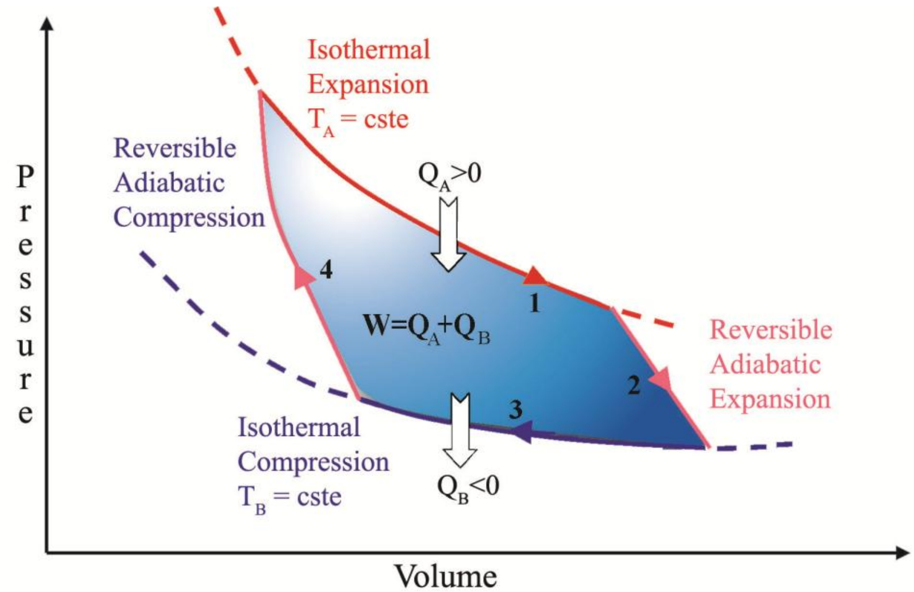

***Figure 4*: The Carnot cycle in the Clapeyron diagram (Pressure/Volume).** *The cycle consists of 2 isotherms and 2 reversible adiabatics: notations 1 to 4 corresponds to the notations above. (Diagram by B. Bradu)*

The Second Law of Thermodynamics

The second law of thermodynamics states that all physical processes such as heat exchange are irreversible processes. A new thermodynamic variable was thus created by Clausius (1822-1888) in 1850 after reading Sadi Carnot's Reflections, in order to show the irreversibility of thermodynamic transformations: the entropy, denoted as "S". This is extensively large (and summative) and defined as follows: its variation during a reversible transformation is equal to the amount of heat given to the system divided by the temperature of this same system:

$$\Delta S = \frac{Q_{rev}}{T} \qquad (1)$$

All actual thermodynamic transformations (also irreversible) create entropy due to dissipative phenomena (friction, diffusion, chemical reactions, etc.). The entropy of a closed system then can only increase over time. The famous inequality of Clausius can be deduced which formalises the second law of thermodynamics and is represented in the inequality:

$$\Delta S > \frac{Q_{irr}}{T} \qquad (2)$$

For the construction of heat engines, it should be understood, with the second law of thermodynamics, that the entropy generation is an obstacle: the entropy created reduces the efficiency of the engines. In other words, engines should be made that minimize the creation of entropy, i.e., mainly friction and heat transfer; which Sadi Carnot had perfectly understood when he says that the first rule to observe in order to design an effective heat engine is to eliminate all heat transfers having an origin other than compression and expansion.

Sadi Carnot in fact states for the first time in *Reflections* the second law of thermodynamics, which would subsequently be formalised by Clausius with entropy 25 years later. By applying the second law to the Carnot cycle which is based on reversible ideal transformations, we can thus write the Clausius-Carnot equality as valid for a reversible Carnot heat engine having a hot source at the temperature T_A and a cold source at the temperature T_B (the entropy is conserved during adiabatic transformations):

$$\frac{Q_A}{T_A} = -\frac{Q_B}{T_B} \tag{3}$$

Carnot efficiency is equal to the ratio between motive power W and the power given to the system Q_A (in step 1, for example by heating water in a steam engine). It depends <u>exclusively</u> on the ratio between the temperatures of the cold source and the hot source, regardless of the vector:

$$\eta_{carnot} = \frac{W}{Q_A} = \frac{Q_A + Q_B}{Q_A} = 1 + \frac{Q_B}{Q_A} = 1 - \frac{T_B}{T_A} \tag{4}$$

Therefore, by using the steam at atmospheric pressure as a hot source (100°C, or 373 K) and the water from a waterway as a cold source (20°C, or 293 K), equation (4) give us a maximum theoretical efficiency of:

$$\eta_{carnot} = 1 - \frac{293}{373} = 21\% \tag{5}$$

All heat engines operating between 100°C and 20°C therefore cannot exceed 21% efficiency. It can be seen in seen in equation (4) that the Carnot efficiency approaches 100% if the cold source is at 0 K, which physics forbids us (this is a consequence of the third law of thermodynamics, formulated later on in 1904 by

Walther Nernst 1864-1941), it is thus impossible to obtain a heat engine having an efficiency of 100%, even with a Carnot heat engine.

Once completing his demonstration, Sadi Carnot reformulates his main proposition in an even more general way on page 38:

> *The motive power of heat is independent of the agents implemented to achieve it; its amount is determined only by the temperatures of the bodies between which the transport of heat, as a last result, takes place.*

It's this last proposition which constitutes truly the start of modern thermodynamics and its application to heat engines. All engineers today working on heat [or combustion] engines, heating systems or refrigerators attempt to come closer to the Carnot cycle in order to obtain a maximum efficiency; but we are still very far from achieving the Carnot cycle industrially. It is easily understood that the greater the difference of temperature between the hot source and the cold source, the more efficient the engine is - because the Carnot efficiency is much higher. For example, refrigeration systems that use liquid helium operate between 300 K and 4.2 K: an ideal Carnot engine would consume, to provide a refrigeration power of 1 W at 4.2 K, a power of 70 W at 300 K[5]; the best helium refrigerators today consume 230 W of electrical power to provide 1 W of refrigeration at 4.2 K. This represents barely 30% of the Carnot cycle, which is mainly due to the entropy generation caused by friction in rotating engines (compressors and turbines) and by the loss of heat during heat transfers (heat exchangers).

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(Translated in English by John Moran, published August 2014)

5. Following formula (3) above, 70/300 ≈ 1/4,2.